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A comparative study on application of Chebyshev and spline methods for geometrically non-linear analysis of truss structures

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ABSTRACT

In this paper, the effectiveness of the modified Chebyshev and cubic spline's iterative methods is comparatively evaluated on geometrically non-linear analysis of truss structures. For the purpose of a comprehensive comparison, we have also proposed an iterative method free from second derivative originated from modified Chebyshev and cubic spline's schemes. The method involves a set of predictor–corrector schemes constructed by Chebyshev as the predictor for spline correctors to improve the approximation of the tangential stiffness matrix. The numerical assessment of the proposed method lies on three-step algorithm with satisfactory convergence of results. The analysis of convergence is carried out and is shown that the proposed method is at least third-order convergent. A simple step-by-step algorithm is developed capable of tracing the non-linear equilibrium curve until the first limit point through an incremental approach. The robustness and efficiency of the proposed schemes are comparatively investigated against classical Newton–Raphson's method for solving practical non-linear problems. It is concluded that for the large structural systems, where a large-scaled stiffness matrix is being iteratively updated, the best computational time, thus the optimum cost of analysis is accomplished by the proposed algorithm using reasonably less number of incremental loads. Finally, it is demonstrated that the proposed procedure and spline's method require considerably less number of iterations to reach the sufficient accuracy.

1. Introduction

In general, the perception and identification of non-linear problems are one of the controversial challenges in different disciplines of science and technology. The procedures of non-linear analysis have attracted much attention due to their computational efficiency, the cost of analysis, feasibility and applicability. One of the most reliable approaches lies on iterative type methods, which are being improved by using Taylor series, quadrature formulation and decomposition. There are many procedures have offered and analyzed different order predictor–corrector algorithms for developing Newton's scheme [1,2]. The main concept is to linearize non-linear equations by developing the non-linear terms with the known solution from former iterations. For instance, the family of Chebyshev and Chebyshev–Halley's methods are improved to construct sufficiently rapid iterative methods [3,4]. The major shortcoming of them is the necessity of second derivatives. To overcome the aforesaid drawback, there are many attempts are delivered to make the predictor–corrector procedure free from the second derivative, however, except for a few of them the rest are incapable for solving $n \times n$-system of equations (such as non-linear analysis of structures) [5,6]. Theoretically, implementation of modified iterative schemes (i.e., the family of modified Chebyshev's method) for assessment of non-linear equations is sufficiently addressed in the literature. It is observed that, they work as a promising tool for non-linear analysis of scalar systems [7,8]. However, the practice of aforementioned algorithms is not well reported in the literature for actual applications.

Moreover, splines are efficient tools for functional interpolation and approximation (in referring to the Runge phenomena, except for the higher degree interpolations). Cubic splines are the most popular ones, where they are smooth and they do not have oscillatory behavior for higher degree polynomial approximation [9]. Theoretically, a third-order iterative method is proposed to solve non-linear scalar problems using a quadrature rule of cubic spline functions [10]. Although, it is shown that they are third-order convergent and satisfactorily reasonable for non-linear scalar systems; in practice, evaluation of the computational efficiency of them for non-linear analysis of structures is not reported.

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Over the past two decades, there have been extensive researches conducted to investigate non-linear performance of frame and truss structures [11–14]. In particular, the non-linear behavior of spatial trusses has been the focus of interest due to easy fabrication, optimum construction and economic points of view. These structural systems may be considered to provide their reliability even after an initial inelastic failure involves a progressive failure that redistributes forces and normally yielding other members to fail until the total structure becomes unstable. To precisely predict and model this progressive failure, the analysis scheme that can construct a load–displacement response during the instability is essential. These procedures concern with creating finite element models that renew their stiffness corresponding to the non-linear geometric response [15]. There are many methods available to trace the equilibrium path governing to the non-linear structures. Newton–Raphson, modified Newton–Raphson, arc-length scheme and the perturbation method [16] are commonly used methods. The normal flow algorithm [17,18], the self-correcting incremental approach and the initial value prediction procedure [19] have been also considered for this purpose.

Analytically, large deformation analysis of truss structures has been evaluated by many researchers. Tabatabai et al. [18] employed the classical Newton–Raphson's approach along the flow path normal in static analysis. Pina et al. [20] presented a formal solution of quasi-static for solving non-linear applications. Tabatabaei and Safari [17] studied large strain analysis of frames using normal flow algorithm. Papadrakakis and Gantes [21] presented some procedures to truncate Newton–Raphson's scheme. Saffari and Mansouri [22] have proposed a fourth-order convergent scheme using two-point method. They have also examined the computational efficiency of their method on large-scaled structures with both geometry and material non-linearity [23]. They concluded that the underlying advantage of two-point method is the rapid convergence, and therefore gaining the less cost of analysis. Greco et al. [24] proposed a new geometric non-linear relation for spatial truss analysis that utilizes the nodal locations rather than nodal displacements. Papadrakakis [25] implemented the dynamic relaxation method for the second-order and large deflection analysis of truss elements. Kassimali and Parsi-Feraidoonian [26] studied the non-linear behavior of pre-stressed cable trusses. In addition, inelastic post-buckling analysis of truss structures by the dynamic relaxation method investigated by Ramesh and Krishnamoorthy [27]. Bellini [28] recommended a new accurate model capable of handling the snap-through and snap-back buckling problems. Thai and Kim [29] introduced the large-deflection inelastic analysis of spatial truss structures including not only geometric but also material non-linearities. Zhang et al. [30] improved a new stable algorithm for the geometrically non-linear analysis of bimodular truss and tensegrity structures originated from the parametric variational principle (PVP) and the co-rotational procedure. They found that the PVP approach had the better convergence behavior for non-linear analysis of these structural systems.

Subsequently, the main objectives of this study involve, (i) to comparatively evaluate the efficiency of Chebyshev and spline's iterative methods in contrast to the commonly utilized Newton–Raphson's method for non-linear analysis of large-scaled structures, (ii) to investigate the applicability and robustness of these methods in non-linear structural analysis, (iii) to propose a straightforward step-by-step algorithm capable of using the proposed algorithms for geometrically non-linear analysis of large-scaled truss structures with the superior computational efficiency of the analysis.

The layout of this paper is structured as follows. A brief review of modified Chebyshev and spline's iterative methods are given in Sections 2 and 3, respectively. The proposed predictor–corrector algorithm using Chebyshev and cubic spline functions are extended in Section 4. Accordingly, the rate of the convergence as well as the efficiency of proposed algorithms is comparatively investigated in this section. In Section 5 of this paper, the conceptual background of geometrically non-linear analysis of structures is briefly presented. Furthermore, the geometric construction and the step-by-step algorithm for the implementation of the proposed methods are discussed in Section 5. Finally, the feasibility and capability of the proposed methods are practically evaluated in Section 6 on four numerical examples.

2. Modified Chebyshev's iterative method

The third-order convergent Chebyshev's method is defined as follows [7]:

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}\frac{f''(x_n) f(x_n)}{f'^2(x_n)}\right)\frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots \tag{1}$$

The major drawback of Chebyshev's method represented in Eq. (1) is the necessity of second derivative. In non-linear analysis, the computation of second derivatives, most of the time is neither possible, nor practical. Therefore, the initial prediction of $y_n = x_n - \theta f(x_n)/f'(x_n)$ for any real and nonzero parameter θ, taking Taylor expansion of $f(y_n)$ about x_n and set of simplifications yields the modified Chebyshev's scheme free from second derivative as follows [7]:

$$x_{n+1} = x_n - \frac{f(y_n) + (\theta^2 + \theta - 1) f(x_n)}{\theta^2 f'(x_n)} \tag{2}$$

It is proven that for variants of θ, iterative procedures stemmed from Eq. (2) are cubically convergent. For instance, using $\theta = 1$ the so-called Potra–Ptak method of the third ordered is obtained as [6]

$$x_{n+1} = x_n - \frac{f(x_n - f(x_n)/f'(x_n)) + f(x_n)}{f'(x_n)} \tag{3}$$

For $\theta = -1$, another scheme is derived as follows [7]:

$$x_{n+1} = x_n - \frac{f(x_n + f(x_n)/f'(x_n)) - f(x_n)}{f'(x_n)} \tag{4}$$

One of the efficient iterative methods is obtained using $\theta = (\sqrt{5} - 1)/2$ as follows [7]:

$$x_{n+1} = x_n - \frac{f\left(x_n - (\sqrt{5} - 1) f(x_n)/(2f'(x_n))\right)}{(3 - \sqrt{5}) f'(x_n)/2} \tag{5}$$

Finally, $\theta = -2$ constructs the next modified Chibyshev's iterative algorithm as follows [7]:

$$x_{n+1} = x_n - \frac{f(x_n + 2f(x_n)/f'(x_n)) + f(x_n)}{4f'(x_n)} \tag{6}$$

To be pointed out that, our concern in this research lies on the modified Chebyshev's methods presented in Eqs. (5) and (6), in which that, they are capable to solve non-linear systems of equations and the best computational efficiency is reported for these two algorithms. Accordingly, in this study the modified Chebyshev's methods designated by CH1 and CH2 refer to Eqs. (5) and (6), respectively.

3. Cubic spline's iterative method

Theoretically, adequate fitting data, smooth behavior and non-oscillatory characteristic are the main advantages of using cubic spline functions for functional interpolation and approximation. The algorithm of iteration of cubic spline functions (designated by SP) involves the initial prediction of Newton $y_n = x_n - f(x_n)/f'(x_n)$. Subsequently, the third-order convergent iterative scheme of SP is developed as follows [10]:

$$x_{n+1} = x_n - \frac{16 f(x_n)}{3f'(x_n) + 10 f'\left(\frac{x_n + y_n}{2}\right) + 3f'(y_n)} \tag{7}$$