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Stresses in finite anisotropic plate weakened by rectangular hole

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ABSTRACT

In this paper, a solution to calculate the stress components around rectangular shaped cutout in a finite anisotropic plate subjected to in-plane loading is presented. The stress functions are derived using complex variable approach and least square boundary collocation method. The influence of plate size, material properties, stacking sequence, hole geometry and loading conditions on the stress concentration is also presented. Some of the results obtained by present method are compared with finite element solutions and with the existing literature.

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1. Introduction

Thin plates of various materials are very commonly used in engineering applications. The different shapes of cutouts are made in plates to cater the need of service/operation. These cutouts exhibit high stress concentration under different loading condition and may cause the catastrophic failure of the components. To understand the catastrophe of the component, it is necessary to have an awareness of the stress concentration around the hole in a plate during design phase.

The closed form solutions for the stresses around different shaped holes in an infinite isotropic and anisotropic plate for different loading have been obtained by Savin [1], Lekhnitskii [2], Ukadgaonker and Rao [3], Rao et al.[4], Sharma [5–7], Sharma et al. [8], Sharma and Dave [9], Patel and Sharma [10], Rezaeepazhand and Jafari [11], Batista [12], Yang et al.[13] and many more using complex variable approach proposed by Muskhelishvili [14]. In all these solutions the load is assumed to be applied at the remote boundary of the plate (i.e. infinite plate) which is not always the case in practical applications like perforated plates, aircrafts windows, automobile windows etc. wherein the loaded boundaries are nearer to the hole and affect the stress distribution around the hole severely. The cases where the boundary are closer to the hole or other way if the ratio of the plate size to the hole size is less than 10, it is considered as the finite plate. The stress distribution around hole in finite plate cannot be estimated through the solution of infinite plate directly.

Some researchers namely Ogonowski [15], Newman [16], Lin and Ko [17], Woo and Chan [18], Madenci et al. [19], Xiwu et al. [20], Xu et al. [21], Zheng and Xu [22] and few others have proposed the solutions for finite plate using complex variable method. The complex stress functions are expressed in terms of infinite power series and the constants of the series are obtained by boundary collocation method. These solutions are limited to circular and/or elliptical hole in isotropic and/or anisotropic finite plate. In many engineering applications the shape of the hole is not only limited to circular or elliptical but the rectangular and polygonal holes are also found its practical importance. The solution for stresses around rectangular hole in finite plate is proposed by Pan et al. [23] by modified stress functions for isotropic material. The literature review suggests that the solution for the stresses around rectangular and square shaped hole in finite laminated plate has not been addressed.

The present work provides a generalized method to obtain the stress distribution around rectangular hole in finite anisotropic plate. Unlike the previous papers, a generalized Schwarz–Christoffel mapping function is used in the formulation of anisotropic finite plate to map the rectangular hole on to the unit circular hole. The Laurent series expansion of the complex stress functions is used and constants of the series are derived using the boundary collocation method. Influence of plate size, hole geometry, material properties and stacking sequence on stress concentration are also studied and presented.

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2. Analytical formulation

A finite sized thin plate having rectangular opening (refer Fig. 1) is subjected to in-plane loading. Applying generalized Hooke's law, Airy's stress functions and strain–displacement compatibility conditions, the stress components in an anisotropic media can be represented in terms of Muskhelishvili's [14] complex stress functions $\phi_j(z_j)$ as

$$\sigma_x = 2\mathrm{Re}\left[\sum_{j=1}^{2} \mu_j^2 \phi_j'(z_j)\right]$$

$$\sigma_y = 2\mathrm{Re}\left[\sum_{j=1}^{2} \phi_j'(z_j)\right]$$

$$\tau_{xy} = 2\mathrm{Re}\left[\sum_{j=1}^{2} \mu_j \phi_j'(z_j)\right] \quad (1)$$

where, $z_j = x + \mu_j y$, μ_j are the complex constants of anisotropy. These constants are roots of the characteristics equation of anisotropic plate (Lekhnitskii [2]) and depend on material properties, fiber orientation and stacking sequence.

2.1. Mapping function

The mapping function to map the area external to the rectangular shaped cutout in z-plane conformally on to the area outside the unit circle in ζ- plane is available in the literature [1] as follows:

$$z = R\left(\zeta + \sum_k \frac{C_k}{\zeta^k}\right) \qquad (k = 2p-1,\ \ p = 1,\ 2,\ 3,\ ....n) \quad (2)$$

where R is the size factor, $\zeta = e^{i\theta}$ andC_k are the constants of the mapping functions. The values of constants of mapping functions are available in the literature for few specific side ratios of rectangle having longer side parallel to X-axis. To produce the rectangle of any desired side ratios (D/d) and also at any orientations (α), the constants of the mapping functions (C_k) are obtained as follows:

$$C_1 = \frac{1}{2}\left(e^{2i(\beta+\alpha)} + e^{-2i(\beta-\alpha)}\right);$$

$$C_3 = \frac{1}{24}\left(e^{2i(\beta+\alpha)} - e^{-2i(\beta-\alpha)}\right)^2;$$

$$C_5 = \frac{1}{80}\left(e^{4i(\beta+\alpha)} - e^{-4i(\beta-\alpha)}\right)\left(e^{2i(\beta+\alpha)} - e^{-2i(\beta-\alpha)}\right); \quad (3)$$

where β characterizes the side ratio of the rectangular hole (refer Table 1) and $\alpha$is orientation angle of rectangular hole with positive X axis as shown in Fig. 1. Table 1 shows the values of these constants for different side ratios of rectangular hole with the corner radius (ρ) and total perimeter. The corner radius is derived using the formula given by Sharma [5].

For an anisotropic media, Eq. (2) is modified by introducing constants of anisotropyμ_j. Due to affine transformation, Eq. (2) takes the form

$$z_j = R\left[a_j\left(\frac{1}{\zeta} + \sum_k C_k\zeta^k\right) + b_j\left(\zeta + \sum_k \frac{C_k}{\zeta^k}\right)\right] \quad (4)$$

where, $a_j = (1 + i\mu_j)/2$, $b_j = (1 - i\mu_j)/2$ and $C_k$are the constants corresponding to the different side ratios of the rectangular hole.

Rearranging the terms in Eq. (4), a polynomial equation of $\zeta$is obtained as

$$\sum_k \left[R a_j C_k \zeta^{2k} + R b_j \zeta^{k+1} - z_j \zeta^k + a_j \zeta^{k-1} + b_j C_k\right] = 0 \quad (5)$$

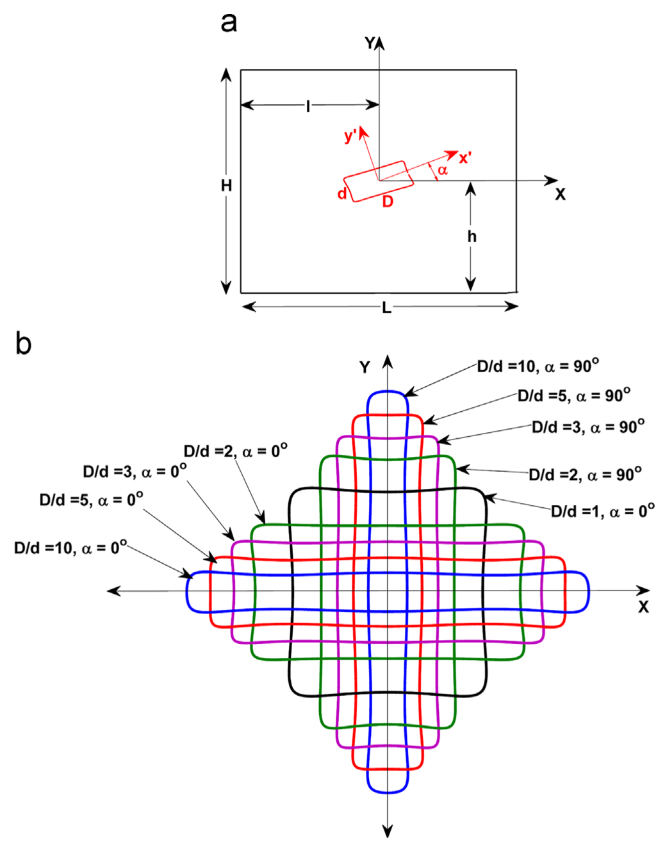

Fig. 1. Geometry of finite plate and rectangular hole.

Table 1
Constants of mapping functions.

D/d $\alpha = 0$	β	C_1	C_3	C_5	Corner radius (ρ)	Total perimeter
1	0.7983	−0.0251	−0.1666	0.0025	0.0996	6.68
2	0.6283	0.3095	−0.1507	−0.0280	0.0679	6.79
3	0.5364	0.4781	−0.1286	−0.0369	0.0548	6.93
4	0.4764	0.5797	−0.1107	−0.0385	0.0554	7.03
5	0.4333	0.6477	−0.0967	−0.0376	0.0597	7.10
6	0.4003	0.6965	−0.0858	−0.0359	0.0649	7.16
7	0.3741	0.7331	−0.0771	−0.0339	0.0710	7.22
8	0.3525	0.7617	−0.0700	−0.0320	0.0761	7.26
9	0.3345	0.7847	−0.0640	−0.0302	0.0808	7.30
10	0.3190	0.8035	−0.0591	−0.0285	0.0848	7.34

The one out of $2k$ roots of Eq. (5) maps the rectangle shape on to the unit circle and it is used for the solution of stress function.

2.2. Stress functions

The stress functions can be taken in the form of infinite power series with negative and positive power terms in ζ_j plane as [2],

$$\varphi_j(\zeta_j) = \alpha_j \ln \zeta_j + \sum_{m=1}^{\infty}\left(A_{jm}\zeta_j^{-m} + B_{jm}\zeta_j^{m}\right), \quad (6)$$

where, α_j, A_{jm} and B_{jm} are unknown constants of the series which are derived from boundary conditions, ζ_j are the mapped

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