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Nonlinear dynamic analysis of Sigmoid functionally graded circular cylindrical shells on elastic foundations using the third order shear deformation theory in thermal environments

Nguyen Dinh Duc [a,*], Ngo Duc Tuan [b], Phuong Tran [b], Nguyen Trong Dao [a], Ngo Tat Dat [a]

[a] Vietnam National University, 144 Xuan Thuy – Cau Giay – Ha Noi, Ha Noi, Vietnam
[b] The University of Melbourne, Parkville, VIC 3010, Australia

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ABSTRACT

This paper presents an analytical approach to investigate the nonlinear dynamic response and vibration of imperfect functionally graded materials (S-FGM) thick circular cylindrical shells surrounded on elastic foundation using the third order shear deformation shell theory. Material properties are assumed to be temperature dependent and graded in the thickness direction according to a Sigmoid power law distribution (S-FGM) in terms of the volume fractions of constituents with metal–ceramic–metal layers. The S-FGM shells are subjected to mechanical, damping and thermal loads. The Galerkin method and fourth-order Runge–Kutta method are used to calculate natural frequencies, nonlinear frequency–amplitude relation and dynamic response of the shells. In numerical results, the effects of geometrical parameters, the material properties, imperfections, the elastic foundations and mechanical loads on the nonlinear dynamic response and nonlinear vibration of the shells are investigated. Accuracy of the present formulation is shown by comparing the results of numerical examples with the ones available in literature.

1. Introduction

Functionally graded materials (FGM) are those in which the volume fractions of two or more materials vary continuously as a function of position along certain direction of the structure to achieve a required function. These novel materials were first introduced by a group of scientists in Sendai, Japan, in 1984 [1,2] and then were rapidly developed by other researchers. Due to their abilities to withstand the extreme thermal mechanical loadings, FGM material have been used extensively for engineering applications. In particular, the FGM cylindrical tubes are utilized for wide ranges of applications such as pressure vessels, oil pipes, nuclear power reactor, automotive components as well as high temperature-resistant components for aerospace engineering applications.

Despite its complexity, the higher order shear deformation theory was used by some researchers to investigate the static stability of FGM thick plates and shells. Mirzavand and Eslami [3–5] investigated the effect of geometrical imperfections on the thermal and mechanical buckling of FGM cylindrical shells under several types of loadings for two different models of initial imperfections. Shen [6] presented a study on the postbuckling response of a shear deformable functionally graded cylindrical shell of finite length embedded in a large outer elastic medium and subjected to axial compressive loads in thermal environments. Shen [7] also investigated the torsional buckling and postbuckling of FGM cylindrical shells in thermal environments. Bahtui and Eslami [8] studied the coupled thermoelastic response of a functionally graded circular cylindrical shell. Wang et al used the Donnell's nonlinear shallow-shell theory and Galerkin method [9] and the method of harmonic balance [10] for investigations of nonlinear dynamic response of rotating circular cylindrical shells with precession of vibrating shape. In Ref. [11], the method of harmonic balance is applied also by Wang et al. to study the nonlinear dynamic response of the muti-degrees-of-freedom system for axially moving laminated circular cylindrical shells. Tung and Duc [12] presented an analytical investigation on the nonlinear response of thick functionally graded doubly curved shallow panels resting on elastic foundations and subjected to some conditions of mechanical, thermal, and thermomechanical loads. Duc and Tung [13] also investigated the buckling and postbuckling behaviors of thick functionally graded plates resting on elastic foundations and subjected to in-plane compressive, thermal and thermomechanical loads. Recently, Zeighampour et al [14] developed the thin shell theory using Hamilton principle and modified strain gradient theory to investigate free vibration of the single-walled carbon nanotube.

Up to date, dynamic analysis of FGM shells using the first-order and high-order shear deformation theories also have received attention of many authors. Beni et al. [15] studied the free vibration analysis of size-dependent shear deformable functionally graded cylindrical shell on the basis of modified couple stress theory. Zhang

* Corresponding author.
E-mail address: ducnd@vnu.edu.vn (N.D. Duc).

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et al. [16] presented a four-unknown shear deformation theory and then develop a shear deformable functionally graded cylindrical microshell model using the strain gradient elasticity theory. Najafizadeh and Isvandzibaei [17,18] studied free vibration of thin FGM cylindrical shells with ring support by using Ritz method based on the first order and higher order shear deformation shell theories, respectively. Matsunaga [19] examined the free vibration and linear buckling of FGM cylindrical shells based on a two-dimensional higher order shear deformation theory. Alibeigloo et al. [20] presented elasticity for the free vibration analysis of functionally graded cylindrical shell bonded to thin piezoelectric layers. Jafari et al. [21] also investigated nonlinear vibration of functionally graded cylindrical shells embedded with penzoelectric layer. Duc et al. [22] proposed an analytical approach on the nonlinear response of thick functionally graded circular cylindrical shells with temperature independent material property surrounded on elastic foundations subjected to mechanical and thermal loads. Tornabene [23] studied the dynamic behavior of moderately thick functionally graded conical, cylindrical shells and annular plates. Bich et al. [24] investigated the nonlinear dynamic response and vibration of imperfect eccentrically stiffened FGM thick double curved shallow shells on elastic foundation using the first order shear deformation theory.

In recent years, the dynamic analysis of FGM shells with temperature-dependent material properties has received the great attention of the researchers. Shen [25] investigated the large amplitude vibration behavior of a shear deformable FGM cylindrical shell of finite length embedded in a large outer elastic medium and in thermal environments. On the other hand, Kadoli and Ganesan [26] calculated the buckling temperature and natural frequencies of FGM cylindrical shells with clamped boundary conditions. In their analysis, the finite element equations based on the first order shear deformation shell theory were formulated. Sheng and Wang [27] performed the analysis on the linear vibration, buckling and dynamic stability of FGM cylindrical shells embedded in an elastic medium and subjected to mechanical and thermal loads based on the first-order shear deformation shell theory. Alijani and Amabili [28] studied the nonlinear dynamic instability of functionally graded plates in thermal environments. Ungbhakorn and Wattanasakulpong [29] investigated thermo-elastic vibration response of functionally graded plates carrying distributed patch mass using the third order shear deformation theory. Alijani et al. [30,31] studied the nonlinear vibrations of FGM doubly curved shallow shells. They considered the thermal effect and used a higher order shear deformation theory. Kadoli and Ganesan [26] obtained the buckling and free vibration analysis of functionally graded cylindrical shells subjected to a temperature – specified boundary condition. Huang and Wang [32] investigated the large amplitude vibration behavior of a shear deformable FGM cylindrical panel resting on elastic foundations in thermal environments. Huang and Han [33] studied the nonlinear buckling of torsion-loaded functionally graded cylindrical shells in thermal environment. Sepiani et al. [34] investigated the free vibration and buckling of a two-layered cylindrical shell made of inner functionally graded and outer isotropic elastic layer, subjected to combined static and periodic axial forces. Sheng and Wang [35] presented the report of an investigation into thermoelastic vibration and buckling characteristics of the functionally graded piezoelectric cylindrical, where the functionally graded piezoelectric cylindrical shell is made from a piezoelectric material having gradient change along the thickness.

However, to the best of the author's knowledge, in spite of its importance, no research work related to the nonlinear dynamic response and vibration of cylindrical shells made of Sigmoid functionally graded materials (S-FGM) shells with metal–ceramic–metal layers under mechanical, damping and thermal loads has been yet presented. Furthermore, there is no any publication using simultaneous combination of the third order shear deformation theory and stress function to investigate the nonlinear dynamic response and nonlinear vibration of the FGM thick circular cylindrical shells.

Recently, Duc and Thang [36] investigated the nonlinear dynamic response and vibration of imperfect eccentrically stiffened functionally graded thick circular cylindrical shells. They used the first order shear deformation shell theory and the shell is subjected only by mechanical loads (without temperature). Unlike in Ref. [36], this paper using the third order shear deformation shell theory and stress function with material properties are assumed to be temperature dependent to investigate the nonlinear dynamic response and nonlinear vibration of the S-FGM thick circular cylindrical shells with metal–ceramic–metal layers and surrounded on elastic foundations. The S-FGM shell is subjected to mechanical, damping and thermal loads. The Galerkin method and Runge–Kutta method are also used for dynamic analysis of the cylindrical shells to give expressions of natural frequencies and nonlinear dynamic response. The results show that the imperfection, elastic foundations, temperature, volume-fractions index and geometrical parameters strongly influence the nonlinear dynamic response and vibration of the S-FGM circular cylindrical shells.

2. Modeling of the S-FGM thick circular cylindrical shell surrounded on elastic foundations

Consider a circular cylindrical shell that is made of the combined ceramic and metallic materials with continuously varying mix-ratios comprising ceramic and metal. The length, mean radius and total thickness of the shell are L, R and h, respectively. The shell is defined in a coordinate system (x, y, z), where x and y are in the axial and circumferential directions of the shell, respectively, and z is perpendicular to the surface and points outwards $(-h/2 \le z \le h/2)$.

The S-FGM circular cylindrical shell is resting on the Pasternak elastic foundation (Fig. 1). The load–deflection relationship of the foundation is assumed to be

$$q = k_1 w - k_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)w \tag{1}$$

where q is the force per unit area, k_1 is the Winkler foundation stiffness and k_2 is the shear subgrade modulus of the foundation.

TD of an arbitrary material property (Pr) may be expressed as [22]:

$$\mathrm{Pr} = P_0\left(P_{-1}T^{-1} + 1 + P_1T^{-1} + P_2T^2 + P_3T^3\right). \tag{2}$$

where T(K) is the temperature, and P_0, P_{-1}, P_1, P_2 and P_3 are some coefficients, $T = T_0 + \Delta T$ and $T_0 = 300$(room temperature).

The material property at any point along the thickness of the FGM cylinder is related through the following equation to the material properties of the constituent materials:

$$\mathrm{Pr}_{eff}(z) = \mathrm{Pr}_m V_m(z) + \mathrm{Pr}_c V_c(z) \tag{3}$$

P_m and P_c are the temperature-dependent properties of the metal and ceramic, respectively, and each may be calculated from Eq. (2); V_m and V_c are the metal and the ceramic volume fractions, respectively.

The volume fractions V_c is assumed by the Sigmoid power-law distribution (S-FGM) [22];

$$V_c(z) = \begin{cases} \left(\dfrac{2z+h}{h}\right)^N, & N \ge 0, \quad -\dfrac{h}{2} \le z \le 0 \\ \left(\dfrac{-2z+h}{h}\right)^N, & 0 \le z \le \dfrac{h}{2}; \end{cases} \quad ; \quad V_m(z) + V_c(z) = 1, \tag{4}$$