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A new approach for modeling of streamlined die operations based on a non-quadratic stream function

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ABSTRACT

Talbert and Avitzure introduced the implementation of a quadratic stream function for derivation of velocity field governing through streamlined die extrusion. In the present study, a non-quadratic stream function is introduced to model various streamlined die processes. In the modified formulation an exponent was designated to the stream function in order to obtain a generalized model. The quadratic stream function approach was restricted to processes with axial flow direction. In contrast, the modified model is capable to study the common forming processes i.e. simple and bimetal extrusion and drawing. The actual velocity field was derived from the generalized field by upper bound technique. The plastic flow was studied through distorted bands and elements that obtained by Eulerian description of motion. Subsequently, the modeling of extrusion and drawing proceed by the calculation of the effective plastic strain distribution. Streamlined die has been designed and proposed based on the fact that it requires less external power than the conventional operations and that deforming material encounters smoother plastic flow. The results revealed the length of die highly influences the deformation mode by affecting the shear component of strain. The capability of the modified stream function to capture various metal forming operations with streamlined die was investigated.

1. Introduction

Metal forming processes such as extrusion, wire drawing and tube drawing through streamlined die requires less external power than the conventional operations and thereby a lot of attention has been drawn to these processes. Some analytical and experimental studies on streamlined die have already been carried out by researchers. Upper bound technique is involved in the most of analytical studies of streamlined die processes. However, they are limited to the computation of external power and never stepped beyond the prediction of normalized pressure and some limited parameters.

In an earlier work, Chang and Choi used the upper-bound solution for tube extrusion through curved dies and studied the effect of die geometry and friction [1]. Yang, et al. proposed a generalized kinematically admissible velocity field for axisymmetric extrusion through curved dies [2]. The proposed method of analysis has made it possible to predict the deformation pattern as well as extrusion pressure. The effect of area reduction and die length has been also discussed in relation to extrusion pressure and deformation.

A few papers representing the study of axisymmetric extrusion through adaptable dies were released by Grdon et al. [3–5]. The authors proposed six kinematically admissible velocity fields and demonstrated that the sine-based velocity field that produces less power is the best one. The authors showed that by fixing the values of two additional constants in the radial flow flexibility function, the shear surfaces diminished. Furthermore, it was revealed that the extrusion through streamlined dies can be modeled without considering the shear surfaces which are constructed due to velocity discontinuity.

A numerical simulation of tube drawing was performed for 316 L stainless steel thin-walled tube by Palengat et al. [6]. They carried out a set of experimental tests in order to acquire mechanical and thermal data during the drawing process. Haghighat and Asgari utilized a generalized velocity field to analyze extrusion process of bi-metallic tubes through dies of any shape [7]. Evaluating the upper bound required force, they reported that the obtained results were well matched with FEM and experiments. In another work, a generalized upper bound technique is applied for the solution of working pressures in extrusion-piercing of hollow tubes [8]. In this study the characteristic of deformation modes of tellurium lead and commercially pure aluminum were experimentally studied. The experimental and theoretical working pressures were compared and a good agreement between the obtained results and other analysis was observed.

In order to derive the velocity field for a certain metal forming operation, several approaches such as Bezier curves [9] and stream function can be utilized. Talbert and avitzure introduced the

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Nomenclature

Ψ	stream function
A_0	cross area of billet
r, x, θ	cylindrical coordinates
A_f	cross area of final product
L	length of die, mandrel and plug
r_m	radius of mandrel
Q	volume rate
r_w	die profile
V_0	entrance velocity
r_c	clamp profile
V_f	exit velocity
r_p	profile of inner surface of core
V_x	axial component of velocity field
σ_c	mean stress flow of core
V_r	radial component of velocity field
σ_s	mean stress flow of sleeve
R_0	entrance radius of die
Δt	duration of a single step
R_f	exit radius of die
α	angle of conical converging die
R_i	inner radius of initial tube
m	friction factor between Deformable and tools
R_{0f}	outer radius of the final tube product
Δv	amount of velocity discontinuity
R_{if}	inner radius of the final tube product
W_i	internal power of deformation
R_{ic}	radius of initial clamp
W_f	frictional power
R_{fc}	radius of final clamp product
$\dot{\varepsilon}_{xx}, \dot{\varepsilon}_{rr}, \dot{\varepsilon}_{\theta\theta}$	normal train rate in the axial, radial and angular directions
R_{pi}	inner radius of initial core
$\dot{\varepsilon}_{rx}$	shear strain rate component
R_{pf}	inner radius of final core product

implementation of stream function for solid mechanic problems and they derived velocity fields of a series of operations [10]. More capability of this technique was also indicated by the investigation of plane strain ECAP carried out by Rejaeian and Aghaie-Khafri [11]. In the most of analytical studies involving streamlined die extrusion, upper bound theorem is presented. However, the authors seldom stepped over the evaluation of the external power and prediction of some parameters such as normalized pressure. In the present study, in addition to the main discussion devoted to the numerical modeling of deformation pattern as well as of strain distribution, upper bound approach was mainly used for the two following reasons.

(i) Among infinite number of kinematically admissible velocity fields the identification of the actual one for developing a detailed study was essential. An actual field minimizes the total power and satisfies physical requirements. Therefore, the utilization of upper bound approach for extremization of the total power is highly effective.

(ii) Most of previous analytical studies for various streamlined die operations are based on upper bound analysis. Thus, in order to validate the present results that are founded on kinematic fields including stream function, velocity field and strain rates, a section of the work was devoted to upper bound approach. Subsequently, the prediction of distortion pattern through streamlined die was carried out using Eulerian description. Then, the effective plastic strain for specified parameters was obtained via strain rate components. So far, for investigation of every single of operations with streamlined die a variety of studies containing different generalized velocity fields have been published. However, present study demonstrates that various streamlined die processes can be analyzed with a single model of stream function.

2. Kinematic model

Talbert and Avitzur proposed a quadratic stream function, Eq. (1), in order to describe the flow of a deforming body in a streamlined die extrusion as illustrated in Fig. 1 [10].

$$\psi = \frac{Qr^2}{2\pi r_w^2} \tag{1}$$

Q and r are axisymmetric volume rate and radial variable, respectively. The function of die profile r_w is presented by:

$$r_w = a + b\cos\left(\frac{\pi x}{L}\right) \tag{2}$$

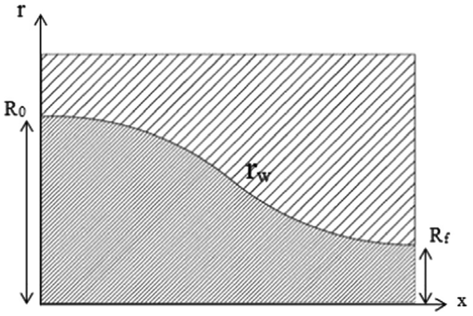

Fig. 1. Schematic diagram and geometrical parameters of axisymmetric extrusion through a streamlined die.

a and b are given as:

$$a = (R_0 + R_f)/2 \tag{3.a}$$

$$b = (R_0 - R_f)/2 \tag{3.b}$$

where L denotes die's length, R_0 and R_f are die's entrance and exit radii, respectively.

Volume rate (Q) for drawing and extrusion can be expressed in term of speed of drawing (V_f) and initial speed of ram (V_0), respectively:

$$Q = A_0V_0 = A_fV_f \tag{4}$$

In this study, a general case which is also capable to describe common metal forming processes with streamlined die (extrusion, wire drawing, tube drawing with mandrel and bi-metal operations) is presented. The incompressible material flow for tube-drawing with a streamlined die is considered as the main element of the study. It will be shown that various streamlined die operations can be analyzed by implementation of negligible changes and different boundary conditions. A non-quadratic stream function for an axisymmetric tube drawing through a streamlined die (Fig. 2) can be expressed as:

$$\psi = \frac{Q}{2\pi}\left(\frac{r}{r_w}\right)^n \tag{5}$$

where n a real number is the exponent of stream function. The exponent is applied to consider all velocity fields satisfying volume constancy requirements. It should be pointed out that the quadratic stream function can be obtained from non-quadratic one when $n = 2$. Thus, quadratic form is a particular case of non-quadratic stream