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Handedness-dependent hyperelasticity of biological soft fibers with multilayered helical structures

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ABSTRACT

Collagen fibrils with multilayered helical structures widely exist in biological soft tissues, e.g., blood vessels, tendons, and ligaments. Understanding the mechanical properties of this kind of chiral materials is not only essential for evaluating the mechanical behaviors of the host tissues but also of significance for medical engineering, clinical diagnosis, and surgical operation. In this paper, a theoretical model is presented to investigate the hyperelasticity of biological soft fibers with multilayered helical structures. The effects of the initial helical angle, number and handedness of the fibers in each ply on the mechanical response of the material are examined. Our analysis reveals a switch of contact modes between two neighboring layers, which may greatly alter the overall non-linear response of the material. The Poisson's ratio of such a multilayered string can be greater than 0.5. The obtained results agree with relevant experiments of soft tissues. This work sheds light on the non-linear mechanics of chiral materials and may also guide the design of biomimetic materials.

1. Introduction

Chiral materials widely exist in biological organisms and tissues. A few examples are spirally striated muscle cells of invertebrates [1], helicobacter pylori [2], spasmoneme of *Vorticella convallaria* [3], flagella filaments of bacteria [4], and fronds of emergent aquatic macrophytes [5]. Tridimensional biological architectures are naturally constructed from lower dimensional chiral materials by, e.g., structural lamination and hierarchy. Of particular interest here are biological soft tissues with multilayered helical structures. Owing to their unique geometric features, these structures may have some unusual mechanical and physical properties and, thereby, enable a series of essential biomechanical functions. For instance, flexible hydrostats such as sea anemones, earthworms, nematodes, echinoderm tube feet, embryonic notochord of frogs, phallus of ducks, and some vermiform animals are generally reinforced by laminated helical fibers, which are also referred to as hydrostatic skeletons [6–11]. Many ectothermic organisms and soft-bodied animals rely on hydrostatic skeletons for the functions of, for instance, support, movement, muscular antagonism, and muscle contraction [12].

Collagen fibrils, the main load-bearing element in a diversity of soft tissues, are ubiquitous in helical forms in, e.g., blood vessels, tendons, ligaments, muscles, skins, and articular cartilages [13,14]. For example, blood vessel walls were modelled as multilayered helical structures consisting of collagen fibers [15,16]. On the basis of experimental observations and measurements [17], Holzapfel et al. modeled the human brain arteries as a two-layered, thick-walled, and circular tube, each layer of which contains a certain number of helical collagen fibers [18,19]. Gasser et al. found that in the media of an artery wall, collagen fibers are arranged into two spirally distributed families [20]. Recently, Flamini et al. divided an aortic wall into six layers and determined the fiber orientation in fresh and frozen porcine aorta by using the diffusion tensor imaging technique [21].

Tendons and ligaments are fibrous, densely connective soft tissues capable of maintaining stability and functions of organ joints [22,23]. The fibers in tendons and ligaments consist of closely-packed collagen fibrils embedded in a proteoglycan rich matrix. It was histologically evidenced that the collagen fibrils in the patellar tendons and anterior cruciate ligaments of both human [24] and canine [25] assume a helical arrangement. By using polarization microscopy, it was observed that the collagen bundles are spirally distributed in both pig chordae tendineae [26] and rat calcaneal tendons [27]. Non-linear constitutive models were established by assuming the collagen fibrils as a cylindrical helical structure [28–30]. Finite element analysis demonstrated that the helical structural model of collagen fibrils is responsible

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for both the non-linear stress–strain behavior and large Poisson's ratios of tendons and ligaments [22].

It is of significance to develop a constitutive model for the highly non-linear mechanical behavior of biological soft tissues reinforced by collagen fibrils [18,31–33]. To illustrate the typical J-shaped stress–strain curve of such soft tissues as skins, the arrangement of their collagen fibrils was generally assumed to be straightened from a disordered to an ordered phase with increasing strain. This empirical explanation ignored the details in the deformation of filaments. To date, the relation between the hyperelastic mechanical response of collagen fibrils and their multilayered helical structures still remain elusive.

In the past decades, much effort has been directed toward exploring the unique geometry and linear elastic property of helical structures [34–50], while few works have addressed their non-linear mechanical behaviors. Wilson and Treloar studied, both theoretically and experimentally, the mechanical properties of yarns consisting of two helical rubber rods [51]. Based on the assumption of continuous stress distribution, Treloar developed a non-linear elastic theory for the multifiber yarns and cords [52]. For rubber yarns consisting of a few fibers, Wilson studied several specific cases where the fibers were arranged in discrete layers [53]. Due to the complexity of the geometric and material non-linearity of multilayered helical structures, there is still a lack of fine theoretical models to predict their hyperelastic behavior.

In this paper, the mechanical behavior of string-like biological soft tissues with multilayered helical structures is theoretically investigated. This paper is outlined as follows. In Section 2, the hyperelastic responses of a straight fiber subjected to axial tension, torsion, and bending are derived. In Section 3, the internal forces and deformations of each fiber in a chiral string are formulated by considering the interaction between neighboring fibers. In Section 4, the effect of microstructural parameters on the mechanical properties of multilayered strings is analyzed. Finally, the main conclusions drawn from this study are summarized.

2. Analysis of a hyperelastic fiber

In this study, we consider strings with multilayered helical structures. For illustration, Fig. 1 shows a straight string consisting of four layers of tightly winded fibers. Assume that all filaments are infinite in length and have a circular cross-section, and the material is hyperelastic, homogeneous, and isotropic. Due to the complex geometry and constitutive relation, it is hard to directly analyze the hyperelastic mechanical behavior of a laminated helical architecture. By comparing the experimental and theoretical results, Wilson and Treloar demonstrated that the finite deformation of a hyperelastic fiber can be decomposed into a few simple cases, including axial tension, torsion, and bending [51]. Therefore, we first investigate the static responses of an individual fiber in these cases, and then the results are used to derive the overall property of a multilayered string. For four representative hyperelastic constitutive models, we provide the explicit expressions of the force–displacement relations.

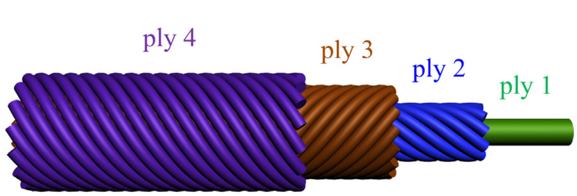

Fig. 1. Model of a string with multilayered helical structures.

2.1. Axial tension and torsion

First consider a straight, cylindrical fiber undergoing large deformation due to coupled axial tension and torsion. Refer to two coincident cylindrical polar coordinate systems $\{X^I\} = \{R, \Theta, Z\}$ and $\{x^i\} = \{r, \theta, z\}$, where $\{\mathbf{G}_I\}$ and $\{\mathbf{g}_i\}$ $(I, i = 1, 2, 3)$ are their covariant basis vectors, respectively. The former coordinate system is used in the reference configuration, while the latter is used in the deformed and current configuration. During the deformation, the fiber keeps straight and its centerline is along the z axis. Let λ_z, λ_r, and $\Delta\tau$ denote its axial elongation ratio, transverse contraction ratio, and axial torsional angle per unit length, respectively. With increasing λ_z, the fiber radius shrinks as $r_f = \lambda_r r_{f^0}$. Throughout this work, the subscript 0 indicates the parameters in the reference configuration. The deformation of the fiber is described by

$$x^1 = \lambda_r X^1,\ x^2 = X^2 + \Delta\tau\lambda_z X^3,\ x^3 = \lambda_z X^3. \tag{1}$$

The covariant and mixed-variable components of the left Cauchy–Green deformation tensor $\mathbf{B} = \mathbf{F}\cdot\mathbf{F}^T$ with respect to the coordinate system $\{x^i\}$ are

$$\left[B^{ij}\right] = \begin{bmatrix} \lambda_r^2 & 0 & 0 \\ 0 & \lambda_r^2 r^{-2} + \Delta\tau^2\lambda_z^2 & \Delta\tau\lambda_z^2 \\ 0 & \Delta\tau\lambda_z^2 & \lambda_z^2 \end{bmatrix}, \tag{2a}$$

$$\left[B^i_j\right] = \begin{bmatrix} \lambda_r^2 & 0 & 0 \\ 0 & \lambda_r^2 + \Delta\tau^2\lambda_z^2 r^2 & \Delta\tau\lambda_z^2 \\ 0 & \Delta\tau\lambda_z^2 r^2 & \lambda_z^2 \end{bmatrix}, \tag{2b}$$

where $\mathbf{F} = \partial x^i/\partial X^I \mathbf{g}_i \mathbf{G}^I$ is the deformation gradient tensor. Using Eq. (2b), one can formulate the principal invariants I_1, I_2, and I_3 of the deformation tensor $\mathbf{B}$ as

$$I_1 = 2\lambda_r^2 + \lambda_z^2 + \Delta\tau^2\lambda_z^2 r^2,\ I_2 = 2\lambda_r^2\lambda_z^2 + \lambda_r^4 + \Delta\tau^2\lambda_r^2\lambda_z^2 r^2,\ I_3 = \lambda_r^4\lambda_z^2. \tag{3}$$

Assume that the material is incompressible, that is, $I_3 = 1$. Therefore, $\lambda_r = \lambda_z^{-1/2}$, and the first two principal invariants I_1 and I_2 are simplified as

$$I_1 = \lambda_z^2 + 2\lambda_z^{-1} + \Delta\tau^2\lambda_z^2 r^2,\ I_2 = 2\lambda_z + \lambda_z^{-2} + \Delta\tau^2\lambda_z r^2. \tag{4}$$

The Cauchy stress tensor $\boldsymbol{\sigma}$ in the fiber is correlated with the deformation tensor $\mathbf{B}$ by the following constitutive relation [54]

$$\boldsymbol{\sigma} = -p\mathbf{I} + 2\left(\frac{\partial W}{\partial I_1}\mathbf{B} - \frac{\partial W}{\partial I_2}\mathbf{B}^{-1}\right), \tag{5}$$

where p is the hydrostatic pressure, $\mathbf{I}$ the second-order unit tensor, and W the elastic strain energy density function. Substituting Eq. (2) into (5) and using $\lambda_r = \lambda_z^{-1/2}$, we obtain

$$\sigma^{11} = -p + 2\left(\frac{\partial W}{\partial I_1}\lambda_z^{-1} - \frac{\partial W}{\partial I_2}\lambda_z\right), \tag{6a}$$

$$\sigma^{22} = -pr^{-2} + 2\left[\frac{\partial W}{\partial I_1}\left(\lambda_z^{-1}r^{-2} + \Delta\tau^2\lambda_z^2\right) - \frac{\partial W}{\partial I_2}\lambda_z r^{-2}\right], \tag{6b}$$

$$\sigma^{33} = -p + 2\left[\frac{\partial W}{\partial I_1}\lambda_z^2 - \frac{\partial W}{\partial I_2}\left(\lambda_z^{-2} + \Delta\tau^2\lambda_z r^2\right)\right], \tag{6c}$$

$$\sigma^{23} = \sigma^{32} = 2\left(\frac{\partial W}{\partial I_1}\Delta\tau\lambda_z^2 + \frac{\partial W}{\partial I_2}\Delta\tau\lambda_z\right), \tag{6d}$$

$$\sigma^{12} = \sigma^{21} = \sigma^{13} = \sigma^{31} = 0. \tag{6e}$$

In the absence of body force, the stress equilibrium requires

$$\frac{\partial\sigma^{11}}{\partial r} + \frac{\sigma^{11} - r^2\sigma^{22}}{r} = 0. \tag{7}$$