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Experimental study of regular and chaotic transients in a non-smooth system

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ABSTRACT

This paper focuses on thoroughly exploring the finite-time transient behaviors occurring in a periodically driven non-smooth dynamical system. Prior to settling down into a long-term behavior, such as a periodic forced oscillation, or a chaotic attractor, responses may exhibit a variety of transient behaviors involving regular dynamics, co-existing attractors, and super-persistent chaotic transients. A simple and fundamental impacting mechanical system is used to demonstrate generic transient behavior in an experimental setting for a single degree of freedom non-smooth mechanical oscillator. Specifically, we consider a horizontally driven rigid-arm pendulum system that impacts an inclined rigid barrier. The forcing frequency of the horizontal oscillations is used as a bifurcation parameter. An important feature of this study is the systematic generation of generic experimental initial conditions, allowing a more thorough investigation of basins of attraction when multiple attractors are present. This approach also yields a perspective on some sensitive features associated with grazing bifurcations. In particular, super-persistent chaotic transients lasting much longer than the conventional settling time (associated with linear viscous damping) are characterized and distinguished from regular dynamics for the first time in an experimental mechanical system.

1. Introduction

Dynamical systems exhibiting discontinuous properties have been a rich area of research [1–16], and the range of possible behaviors in the vicinity of bifurcations is carefully described in [17–19]. Traditional studies tend to focus on steady-state behavior and local bifurcations, and the vast majority of such work has been based on analysis and numerical simulations. This paper carefully examines practical aspects of transient dynamics, and uses an experimental impacting mechanical system as a testbed to distinguish different classes of behaviors that can occur. We also explore the operational differences in finite time-series data between super-persistent transients and what might conventionally be interpreted as steady oscillations.

Because relatively few experimental studies have been conducted on chaotic transients in non-smooth systems, this paper will shed light on systems exhibiting grazing bifurcations and super-persistent chaotic transients [20]. A more thorough experimental protocol will be demonstrated to detect the isolated remote attractors, that is, behaviors not ordinarily revealed by the quasi-static continuation (sweeping up/down) of a system with respect to a bifurcation parameter.

Monitoring the behavior of transients induced by small perturbations is the cornerstone of linear stability theory. Here, we extend this notion to include large perturbation-induced transients (more properly termed disturbances) to augment the more familiar bifurcation diagram [21]. Generating an ensemble of initial conditions can be challenging in experimental studies but is shown to be a very valuable element for exploring the global behavior in the system.

2. Experimental system

Our experimental system consists of a hard rubber ball (diameter 63 mm, weight 145 g) attached to a steel rigid-arm pendulum whose pivot is mounted on a horizontal shake table [22,23]. The motion of the pendulum was recorded as time series of the angular position, $\theta(t)$. A rigid wall is placed such that at an angle of $\theta_{\text{wall}} = -30^{\circ}$, the mass at the end of the pendulum arm comes in

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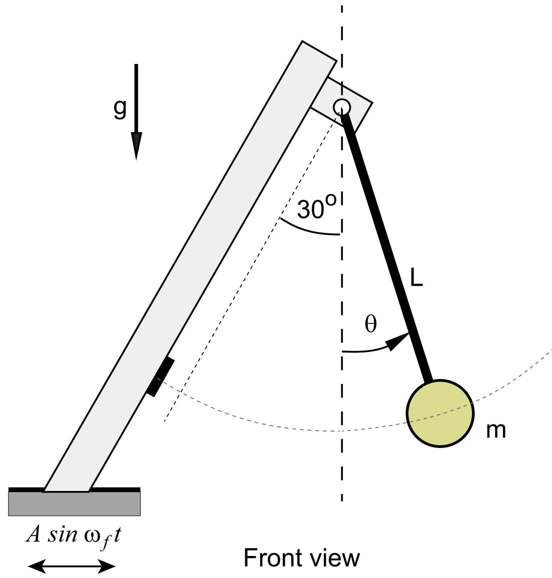

Fig. 1. Geometry of the pendulum system: front view, orthogonal to the plane of forcing.

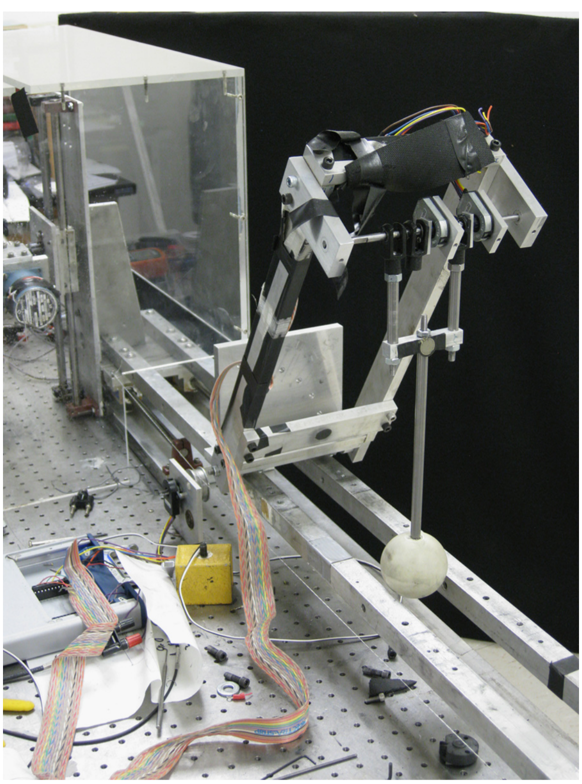

Fig. 2. The experimental system. The Scotch-yoke forcing mechanism (rear) drives the motion of the pendulum along parallel rails (foreground).

contact with this fixed barrier. Schematic and photographic images of this system are shown in Figs. 1 and 2 respectively.

The apparatus is attached to a Scotch-yoke forcing mechanism [24] capable of supplying a harmonic excitation to the base over the range of forcing frequencies $\omega_f = 0.2$ to 2 Hz.[1] The system was designed so that the natural frequency for resonance of the linearized low-amplitude motion of the pendulum is in this range ($\omega_n \approx 0.96$ Hz). This range gives us access to the broad array of non-linear large-amplitude behavior involving impacts occurring beyond the fundamental grazing bifurcation, where $\theta = \theta_{\text{wall}}$ is first achieved.

The motion of the pendulum arm is measured to high accuracy using a quadrature encoder. A tachometer is used to extract the instantaneous position of the pendulum at a given forcing phase. The position at a quarter-cycle later, $\tau = 1/(4\omega_f)$, is also extracted, since subsequent results utilize time-lag embedding, a convenient means of recreating the phase trajectories as well as defining a Poincaré section – an especially useful means of examining dynamic behavior [25].

3. Mathematical model

The system is driven by the imposed horizontal motion of the base, given by $x_f(t) = A_f \sin(\omega_f t)$, where A_f, ω_f are respectively the forcing amplitude and frequency. Then the governing equation of motion for the system can be obtained from Hamilton's principle with an energy dissipation term to model the damping due to friction [23],

$$\frac{d^2\theta}{dt^2} + \omega_n^2 \sin(\theta) + \frac{A_f \omega_f^2}{\ell} \sin(\omega_f t + \phi)\cos(\theta) = -D(\theta, \dot{\theta}), \tag{1a}$$

where ℓ is the effective length of the pendulum and ω_n is the natural frequency based on this length and the gravitational acceleration, $\omega_n = \sqrt{g/\ell}$.

Damping, D, in (1a) is modeled by contributions from linear viscous friction and Coulomb friction [26], both assumed to be acting at the pendulum's pivot,

$$D(\theta, \dot{\theta}) = 2\zeta\omega_n\dot{\theta} + \mu\left(\dot{\theta}^2 + \omega_n^2 \cos\theta\right)\operatorname{sgn}(\dot{\theta}), \tag{1b}$$

where ζ and μ are friction constants.

Because of the fixed rigid barrier, the motion of the pendulum is limited to $\theta(t) \geq -30^\circ$, see Fig. 1. For small $\theta(t)$ ($|\theta| < |\theta_{\text{wall}}|$) there is no impact and the motion is fully determined by (1ab) and will follow the smooth dynamics of a damped driven pendulum. Under forcing yielding $\theta = \theta_{\text{wall}}$, impacts occur, with the minimal case having zero-velocity impacts at *grazing bifurcations*. Impacts of the pendulum with the wall at time t ($\theta(t) = \theta_{\text{wall}}$) are treated as instantaneous inelastic collisions with coefficient of restitution, $0 \leq k < 1$, such that the state immediately after impact is given by $\theta(t^+) = \theta_{\text{wall}}$ with

$$\dot{\theta}(t^+) = -k\dot{\theta}(t^-). \tag{1c}$$

Values for the system parameters were measured from experimental data and are assumed to be constant with respect to time, see Table 1.

While it is reasonable to question whether the damping parameters might evolve slowly with wear and aging of the system and generation of heat during long experiments, we argue that these variations can be neglected based on the excellent agreement that

Table 1
System parameters.

Parameter	Description
$\ell = 26.5$ cm	Pendulum arm effective length
$\omega_n = 0.9688$ Hz	Linear natural frequency
$\zeta = 2.1 \times 10^{-2}$	Linear viscous damping ratio
$\mu = 6 \times 10^{-3}$	Non-dimensional Coulomb damping coefficient
$\theta_{\text{wall}} = -30^\circ$	Position of the fixed barrier
$k = 0.557$	Non-dimensional coefficient of restitution
$A_f = 6.35$ cm	Non-dimensional amplitude

[1] Note that we use Hertz for all frequencies rather than a non-dimensional form.