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Birth of periodic and artificial halo orbits in the restricted three-body problem

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A B S T R A C T

We investigate the bifurcation of artificial halo orbits from the Lyapunov planar family of periodic orbits around the collinear libration points of the circular, spatial, restricted three-body problem. Beside the gravitational forces, our model includes also the effect of the Solar Radiation Pressure (SRP) and this motivates the use of the term 'artificial' halo orbits. Indeed, as a typical problem, one may think of a solar sail, which is characterized by a performance parameter measuring the strength of the effect of the SRP on the spacecraft.

To settle the model, we determine the position of the collinear points as a function of the mass and performance parameters and the energy values at which Hill's surfaces allow for transit orbits between the primaries. To analyze the dynamics we use a consolidated procedure which consists in the computation of a resonant normal form, allowing the reduction to the center manifold and providing an integrable approximation of the Hamiltonian dynamical system. Finally, we compute the bifurcation thresholds of the 1:1 resonant periodic orbit families (which have the standard 'halo' orbits as their first member) as a function of the performance and mass parameters.

The results show that SRP is indeed a relevant ingredient for new dynamical features and must definitely be considered when planning a mission of a solar sail with trajectories in the neighborhoods of collinear points.

1. Introduction

Since the works of Euler [6] and Lagrange [12], it is known that the restricted three-body problem admits five equilibrium points in the synodical reference frame (namely, a frame rotating with the angular velocity of the primaries). Three of such equilibrium positions, named the *collinear* equilibria, are located along the line joining the primaries and are shown to be unstable, while the other two equilibria, called *triangular* positions, are stable provided the mass ratio of the primaries is lower than a given threshold (see, e.g., [15]).

The aim of this work is to investigate the effect of the Solar Radiation Pressure (SRP hereinafter) on the existence of periodic orbits around the collinear libration points of the circular, spatial, restricted three-body problem, and precisely the 1:1 resonant periodic orbit families (which have the standard so-called *halo* orbits as their first member). These orbits are three-dimensional periodic trajectories resulting from the interaction between the gravitational pull of two planetary bodies, and the Coriolis and centrifugal accelerations acting on the spacecraft. They bifurcate/annihilate from/to the Lyapunov orbits with bifurcation sequences parametrized by the energy with thresholds determined by the two relevant parameters of the model, the mass-ratio of the primaries μ and the SRP performance parameter β.

The dynamics around the collinear points has gained an increasing interest in the space era. Since then, several space missions have fully exploited the capabilities of such equilibrium positions. Furthermore, it was suggested to use the Earth–Moon L_2 halo orbit as a communication relay station for an Apollo mission to the far side of the Moon, as it would enable continuous views of both the Earth and the hidden Moon. Yet, the establishment of a bridge for radio communication is a significant problem for future space missions, planning to use the outer side of the Moon as a launch site for space explorations or as an observation point.

Moreover, a number of missions have used the Sun–Earth L_1 halo orbits, like the International Sun–Earth Explorer (ISEE-3 1978), the Solar and Heliospheric Observatory (SOHO 1996) and Genesis (2001). All these space missions have a strategic importance for solar-wind physics, cosmic-ray physics, and astrophysics.

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Remarkably, the Next Generation Space Telescope (NGST) and Lisa Pathfinder will also use halo orbits.

There are extensive results in the literature about the determination of accurate approximations of such equilibrium orbits. Just to quote some results, in 1973 Hénon [10] studied the stability of the planar Lyapunov orbits with respect to vertical perturbations, see also [16]. A center manifold reduction was used by Barden and Howell [1], Jorba and Masdemont [11] and Gomez and Mondelo [9] in combination with the Lindstedt–Poincaré method, which enabled them to develop a semi-analytical technique to describe and compute solutions in the extended neighborhood of an equilibrium point. A method for the analytic evaluation of the bifurcation thresholds in terms of the energy in the rotating frame has been progressively illustrated in [5,2,3], the latter work showing a good agreement with the numerical results found in the literature.

Using the same methodology developed in the above-mentioned works, this paper extends the results to the case accounting for the effect of SRP into the model. The effect of SRP implies that the position of the collinear points will be slightly modified. In addition, Hill's regions are altered by the SRP (compare with Section 2). As it is well known, there exists an energy range in which the region of admissible motion is confined around each primary, preventing transfers between them. By modifying the energy, it is possible to open the gates at the Lagrangian points L_1, L_2 and L_3 in sequence, thus enabling transit and escape orbits, respectively. The effect of the SRP will be to increase the energy threshold at which the gates open, when compared with the model without SRP. The study of the location of the collinear points as well as of the corresponding Hill's regions will be performed, for completeness, for all three collinear points, although the analysis of the bifurcation thresholds will be limited to L_1 and L_2, since the equilibrium point L_3 has no relevant applications in space dynamics; furthermore, it has been shown in [3] that the normal form is not a reliable technique for L_3, when the mass ratio of the primaries is smaller than 10^{-2}, since the optimal order of expansion is very low (see [3] for full details).

Thus, limiting ourselves to L_1 and L_2, we show that exploiting SRP with the use of a reflecting device, e.g. a solar sail, it is possible to get a change of the energy thresholds at which halo orbits and other periodic orbits take place in the vicinities of L_1 and L_2. The results show that SRP significantly affects the energy needed for the bifurcations of the periodic orbits, especially for low mass ratios. Moreover, we provide the thresholds for the bifurcations of other families of periodic orbits, and their behavior as a function of the mass and sail parameters. Indeed, around the equilibria, SRP enables bifurcations of other families of periodic orbits, lowering their bifurcation thresholds to conceivable and reachable values.

This paper is organized as follows. Section 2 provides the equations describing the dynamics of the spatial, circular, restricted three-body problem (hereafter, SCR3BP) with solar radiation pressure. Moreover, we describe a procedure to derive an explicit formulation for the position of the collinear equilibria in terms of the mass and solar sail parameters. Finally, we compare the energies of the zero-velocity curves for the cases with and without SRP. Section 3 provides the fundamental steps to reduce the Hamiltonian to the center manifold; however, since the procedure has been inherited by previous works, only the main steps are hereby described. The values for the bifurcation of the resonant periodic orbits with SRP around L_1 and L_2 are derived in Section 4, where the behavior of the thresholds is analyzed in terms of the mass parameter, and for a range of physically relevant values of the performance parameter. Some conclusions are given in Section 5.

2. Collinear points in the three-body problem with SRP

In this Section we introduce the equations of motion describing the SCR3BP and we present a model including the effect of solar radiation pressure. Within such framework we determine the position of the collinear equilibria, taking care of the dependance of their position upon the mass ratio of the primaries and the *solar sail* performance parameter (namely, the SRP parameter). A first analysis of the energy levels characterizing the system is carried on using the zero-velocity curves. These curves first confine the admissible motion around the primaries (or in the outer space), then, increasing the energy, first allow the planetary interchange orbits and then escape through L_2 and L_3. In particular, we study the dependence of such energy levels on the SRP parameter. As we mentioned in Section 1, we will discuss all three collinear points, although in Section 4 the discussion will be limited to L_1 and L_2.

2.1. The model

We consider the dynamics of a massless body, moving under the gravitational attraction of two massive bodies, say P_1, P_2, called the primaries. We assume that the primaries move on circular orbits with constant angular velocity around their common center of mass. The biggest primary is supposed to be a radiating body, while the massless body is assumed to be a perfectly reflecting solar sail (see, e.g., [14]), which is characterized by a *performance parameter*, say β, defined as

$$\beta \equiv \frac{L_\odot Q}{4\pi c\, GM_\odot B}. \tag{1}$$

The quantities appearing in (1) have the following meaning: $L_\odot = 3.839\times 10^{26}$ W is the Sun's luminosity, $Q\equiv 1+c_R$ where c_R is the reflectivity coefficient of the sail, c is the speed of light, G is the gravitational constant, $M_\odot$ is the mass of the Sun and $B=m/\mathcal{A}$ is the mass-to-area ratio of the spacecraft (m is the mass and $\mathcal{A}$ is the area of the spacecraft). We refer to [8] for a model encompassing a non-perfectly reflecting sail.

We consider a synodic reference frame (O,X,Y,Z) with origin O located in the center of mass of the two primaries; the frame rotates with their angular velocity, so that the positions of the primaries are fixed on the X-axis, the Y-axis belongs to the plane of motion of the primaries and the Z-axis is chosen so that (O,X,Y,Z) forms a clockwise oriented frame.

We scale the units of measure such that the sum of the masses of the primaries, their distance and the angular velocity is set to unity. Let μ and $1-\mu$ be the scaled masses of the primaries with $\mu\in(0,1/2]$. Then, the position of the smaller primary is at $(-1+\mu,0,0)$, while the larger primary is located at $(\mu,0,0)$. With such convention L_2 is at the left of the smaller primary, L_1 is between the primaries and L_3 is located at the right of the larger primary.

Let (X,Y,Z) be the coordinates of the third body in the synodic reference frame and let (P_X,P_Y,P_Z) be the conjugated kinetic momenta defined as $P_X=\dot X-Y$, $P_Y=\dot Y+X$, $P_Z=\dot Z$. We assume that the solar sail is perpendicular to the Sun-sail direction; notice that this assumption ensures the Hamiltonian character of the model. We refer to [7] for different models in which the orientation of the sail is varied.

With these notations and settings, the equations of motion are given by

$$\ddot X - 2\dot Y = \frac{\partial\Omega}{\partial X}$$
$$\ddot Y + 2\dot X = \frac{\partial\Omega}{\partial Y}$$