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Dynamical instability of laminated plates with external cutout

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ABSTRACT

A method to study dynamical instability and non-linear parametric vibrations of symmetrically laminated plates of complex shapes and having different cutouts is proposed. The first-order shear deformation theory (FSDT) and the classical plate theory (CPT) are used to formulate a mathematical statement of the given problem. The presence of cutouts essentially complicates the solution of buckling problem, since the stress field is non-uniform. At first, a plane stress analysis is carried out using the variational Ritz method and the R-functions theory. The obtained results are applied to investigate buckling and parametric vibrations of laminated plates. The developed method uses the R-functions theory, and it may be directly employed to study laminated plates of arbitrary forms and different boundary conditions. Besides, the proposed method is numerical-analytical, what greatly facilitates a solution of similar-like non-linear problems. In order to show the advantage of the developed approach, instability zones and response curves for the layered cross- and angle-ply plates with external cutouts are constructed and discussed.

1. Introduction

Multilayer plates and shells made of composite materials are widely used in many branches of industry and civil engineering. Usually, laminated composite structures have cutouts which are introduced due to industrial and practical requirements. Note that these cutouts may appear as either internal or external ones. The ways to fix these cutouts are varied and depend on the destination and the location of holes or cutouts. It is well known that, as a rule, cutouts change static and dynamic characteristics of the structures which are subjected to various time-dependent external loads. Therefore, a study of a dynamic behavior of multi-layer elements of thin-walled structures with cutouts is one of important problems arising in engineering design and fabrication.

The described problem has been investigated in numerous publications [1,2,5,6,15–17,19,23,24]. An overview of the references devoted to an analysis of laminated plates with cutouts is given by Anil et al. [2], Datta and Biswas [8], Qatu et al. [19], Sahu and Datta [24,25], Zhang and Yang [26], Chen et al. [27], and others.

Our critical review of the state-of-the-art of the problem reported in the available publications and devoted to the study of parametric vibrations of laminated plates with inhomogeneous subcritical states have shown that rectangular plates with a free square or circular central cutout have been mostly considered [2,6,17,23,24,26]. Boundary conditions on the outer contour are usually either simply supported or clamped. In order to study the non-linear dynamics of plates with cutouts, the majority of authors suggest applying the finite element method (FEM) [5,10,18,26]. It should be noted that there are practically no publications devoted to an investigation of plates having complex shape and various arrangement of holes or cuttings. However, in practice, thin-walled composite elements of different shapes and with various types of cutouts are commonly used. In addition, ways of fastening and loading may be also varied while manufacturing plates of complex shapes. These observations, coming from industrial requirements, yielded the motivation of our paper.

To investigate parametric vibrations of laminated plates of a complex form, original and novel approach has been proposed earlier in our works [3,12,13]. The approach has been based on an application of the R-functions theory [22] and variational methods, which makes it applicable to study vibrations of plates with cutouts of different forms and having different boundary conditions. The mentioned method has been developed for thin laminated plates, for which the classical plate theory (CPT) is validated. In addition, the method has been successfully applied to plates under counterbalancing compressive loads. In the present study, the abovementioned approach is generalized for moderately thick laminated plates, parametric vibrations and stability of which can

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be analyzed by the refined theory of the first order (FSDT). We study laminated plates, provided that their subcritical state may be inhomogeneous, i.e. a plate can have external or internal cutouts as well as have different ways of their fastening. Besides, acting loads can balance each other through a clamped part of the boundary. It should be emphasized that, according to the proposed approach, it is necessary to solve a series of linear boundary value problems. These problems are solved by the Ritz method combined with the R-functions theory [11,22]. Due to original representation of unknown displacements and application of the Bubnov–Galerkin procedure, the initial non-linear system of PDEs (partial differential equations) governing the plate dynamics is reduced to a non-linear system of ordinary differential equations (ODEs) of a second order. The coefficients of this system are obtained in an analytical form for plates of an arbitrary shape and expressed by double integrals of constructed functions.

It should be emphasized that a numerical realization of the developed algorithm for a multi-mode approximation, and also for solving the obtained non-linear system of ODEs consisting of numerous equations of the second order, belongs to challenging tasks in the case of domains with complex forms. However, for many engineering applications it is sufficient to consider a single-mode approximation, which requires a study of the classical Mathieu non-linear equation. As it has been shown in many studies reported in Refs. [28–36], this equation is used to predict non-linear phenomena yielded mainly by a parametric resonance. Motivated by this recently revisited application of the non-linear Mathieu equation, our analysis has been restricted to only the first mode approximation. Application of the developed numerical-analytical approach has allowed to construct instability zones as well as to study non-linear vibrations after a loss of stability. The effects of various parameters on the regions of dynamic stability/instability have been illustrated and discussed using a few computational examples.

On the other hand, the presented results obtained for non-linear vibrations of laminated plates by means of application of a single mode can be considered as a solution to the problem in the first approximation. In a general case, when a multi-mode interaction plays a crucial role, a convergence study with an increasing number of modes should be carried out. However, the numerical realization of the latter problem is quite complicated in the case of complex geometry domains.

Since our numerical examples are restricted to only a first mode approximation, we briefly describe our motivation for this study, in particular from the point of view of engineering applications. Recently, high interest in manufacturing sensors and actuators using electrically-actuated continuous members like microbeams and microplates (the so-called MEMS devices and systems) is observed in a wide spectrum of mechanical and mechatronics industries. The existing manufacturing techniques and infrastructure of semiconductor industry, introducing new design and ways of testing performance of microbeam-based and microplate-based systems, are mainly aimed at the reduced-order models of simple one-degree-of-freedom lumped-mass torsional or translational systems.

The classical well-known second-order non-linear ODEs are widely applied in the field of MEMS. In the paper by Turner et al. [28], the Mathieu equation has been utilized for modeling dynamics of a swing, the stability of ships and columns and Faraday's surface wave patterns on the water, putting emphasis on parametrically excited torsional oscillators in a single-crystal silicon MEMS system.

A torsional silicon microresonator has been fabricated and studied in Ref. [29]. It consisted of a rectangular paddle symmetrically suspended by narrow beams of 150–200 nm in width. These metallic layers have been evaporated on the top surface introduce an electric input. Motion of the MEM structure is caused by applying potential between the top and bottom surfaces. A simple second-order ODE has been used to model behavior of continuous systems. In particular, an evidence of parametric amplification has been illustrated and analyzed by modulating the spring constant at twice the resonant frequency.

An auto-parametric amplification-based continuous mass sensor consisting of parallel interdigitated comb finger banks and two sets of non-interdigitated comb fingers on each side, supplemented with four folded beams providing elastic recovery forces, has been modeled, analyzed and experimentally validated by Zhang et al. [30]. The non-linear Mathieu equation has been applied to predict non-linear phenomena exhibited by the main parametric resonance.

The Ruelle–Takens route to chaotic resonance of a nanomechanical device manufactured of three coupled beam resonators excited by magnetomotive driving has been reported by Scheible et al. [31]. The experimentally obtained Fourier spectrum has motivated the authors to apply the standard Duffing equation to study the system chaotic response, although the authors have reported a need to introduce more degrees of freedom.

Zhang and Meng [32] have introduced the simplified mass-spring-damping model of an electrostatically actuated micro-cantilever MEMS device. In particular, main and parametric resonances as well as non-linear behaviors, when subjected to different applied voltages, cubic non-linearity and non-linear damping, have been analytically studied both by means of the harmonic balance method and numerically. Deformations of two plates with a constraint have been omitted while modeling the micro-cantilever device as a system of one degree of freedom. Linear and non-linear tuning of parametrically excited designed and fabricated MEMS oscillators as well as chaos of the MEMS oscillators have been studied in Refs. [33,34]. In both cases, simple non-linear oscillators governed by non-autonomous second-order ODEs with either constant or time periodic coefficients are used.

Owing to the overview of the state-of-the-art, let us emphasize now the importance of a single-mode approximation while studying the behavior of microbeams/microplates in MEMS devices and beyond. A direct use of FEM (Finite Element Method) or FDM (Finite Different Method) is expensive and time consuming. These sophisticated approaches are applied to study performance of already fabricated devices, rather than to design feedback control laws while manufacturing a MEMS device. The engineers expect simple rules and models for designing a MEMS system, which is contrary to the FEM/FDM approaches mostly using numerous variables to describe both the dynamical state of the investigated systems and complexity of the design space.

Therefore, the so-called reduced-order models or macromodels [35] are highly required to capture the most important characteristics of MEMS devices or other continuous mechanical systems employing only a few variables governed by linear/non-linear ODEs. However, on the contrary to purely engineering approaches, we offer a novel technique based on the FSDT and CPT matched with the variational Ritz method and the R-functions theory. As a result, the reduced-order models are yielded by advanced mathematical modeling. Presented models are efficient, accurate and allow to get accurate results relatively fast.

The paper is organized in the following way. The problem statement is defined in Section 2, including the governing differential equations. The method of solution is described in Section 3, whereas numerical examples are presented in Section 4 consisting of the validation study and the estimation of the instability zones of the studied plates. Concluding remarks (Section 5) finish our investigations reported in the paper.