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An eigenanalysis-based bifurcation indicator proposed in the framework of a reduced-order modeling technique for non-linear structural analysis

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ABSTRACT

The Koiter–Newton method is a reduced order modeling technique which allows us to trace efficiently the entire equilibrium path of a non-linear structural analysis. In the framework of buckling the method is capable to handle snap-back and snap-through phenomena but may fail to predict reliably bifurcation branches along the equilibrium path. In this contribution we extend the original Koiter–Newton approach with a reliable and accurate bifurcation indicator which is based on an eigenanalysis of the reduced order tangent stiffness matrix. The proposed indicator has a negligible numerical effort since all computations refer to the reduced order model which is typically of very small dimension. The extension allows the identification of bifurcation points and a tracing of corresponding bifurcation branches in each sector of the equilibrium path. The performance of the method in terms of reliability, accuracy and computational effort is demonstrated with several examples.

1. Introduction

In aerospace engineering, many structures are prone to be limited in their load carrying capability by buckling, while staying in the linear elastic range of the material [1–4]. It is very important to trace the complete equilibrium paths, i.e. load-displacement curves, in order to know and to comprehend the characteristics of structures subjected to loads [5–8]. For structural buckling two main phenomena are generally recognized, namely limit-point buckling and bifurcation-point buckling. The instability properties of some structures may even include both buckling phenomena. An accurate assessment of buckling is highly significant in the structural design, since the load carrying capability of a structure is largely overestimated if a bifurcation point that lies before the limit point is missed during the non-linear path-following analysis. The consideration of possible bifurcations during the path tracing of the non-linear response of a structure includes two key features, the detection of bifurcation points and the branch-switching from one equilibrium path to another equilibrium path of the structure. The implementation of the two features into general path-following methods has created a number of available numerical techniques for bifurcation analysis.

Non-linear path-following techniques, e.g. the arc-length method and its variants [9–11], work well for applications where pure limit-point-type buckling with a unique equilibrium path is present, but have difficulties to converge to bifurcation points and generally prevent an automatic switch from the pre-instability path to one of the bifurcation paths. In general, a small physical imperfection is introduced to allow a switch of the primary equilibrium path to a neighboring bifurcated equilibrium path [1,2,12]. The approach bears the risk to influence the path prior to the bifurcation point with changed characteristics of the considered structure. Another, more accurate possibility to allow for a branch switch at the bifurcation point is the consideration of a suitable fictitious perturbation added to the equilibrium path near bifurcation points [13].

A modification of arc-length-type methods for reliable bifurcation analysis has been in the focus for many years and is repeatedly addressed by many researchers. One popular procedure [9,10] to detect the bifurcation point is to check the sign of the determinant of the tangent stiffness at the current calculation step which changes if either a limit point or a bifurcation point is passed. However, the tracing analysis after the bifurcation point cannot continue unless some further, usually computationally expensive, analyses are taken into consideration. Similarly, the

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sign of stiffness parameters and eigenvalues of the tangent stiffness matrix of the current analysis step are often used to analyze the direction of the equilibrium path [14]. Feng et al. [15–17] took the history of the current equilibrium path into consideration to produce a reliable path direction prediction even in the presence of bifurcations. Eriksson [18] used the scalar product of two path-related measures, the critical eigenvector and reference load vector, to have a reliable bifurcation indicator which well distinguishes between bifurcation and limit points. To achieve a branch-switching, Wagner and Wriggers [19] added the critical eigenvector as a perturbation to the primary path to shift one of the possible bifurcation branches. Recently, Zhou et al. [13] proposed an arc-length method combined with an eigenanalysis-based branch-switching method to discover additional bifurcation points and corresponding secondary paths of cylindrical panels. The critical eigenvector obtained from the eigenanalysis is commonly used in the aforementioned computational procedures to allow a tracing of the bifurcation branch after the bifurcation point which requires the critical eigenvector to be updated in each iteration step. However, it is not practical to do the eigenanalysis of the stiffness matrix in each iteration step in the view of the tremendous computational cost, especially for large-scale finite element models. Noguchi [20,21] has proposed an innovative eigenanalysis-free idea to extract the buckling mode for bifurcation instability from the LDL^T-decomposed stiffness matrix. Wardle [22,23] proposed an asymmetric meshing technique(AMT) to deal with the case that the bifurcation buckling in asymmetric mode exists before the first limit point on the equilibrium path.

The reduction method based on the Rayleigh–Ritz or perturbation techniques [24,25], which can reduce the number of degrees of freedom in non-linear mechanics, is a valuable alternative for structural buckling analysis. Some of the reduction methods [26–30] work in combination with path-following techniques to trace the entire non-linear equilibrium path based on a predictor-corrector approach. The prediction is achieved using an asymptotic expansion [24,25,30] to reduce the number of unknowns in the original equilibrium equations. The solution of the reduced equations is regarded as a predictor for the real equilibrium path. These methods often use the path derivative [28,30] as the path parameter in the expansion of the equilibrium equations, which causes difficulties near bifurcation points. Boutyour, Vannucci and Jamal [31–33] have proposed a bifurcation indicator to detect bifurcation points in the framework of the Asymptotic Numerical Method (ANM) [24,34,30]. After introducing a fictitious perturbation force to the structure at a point of the solution branch, the bifurcation indicator is expressed as a scalar function to measure the intensity of the system response to perturbation forces. The value of this indicator is evaluated through the path-following procedure and becomes zero exactly at the bifurcation point. Recently, the bifurcation indicator that was proposed by Boutyour et al. [33] has been successfully implemented in various mechanical problems of thin films on compliant substrates to detect multiple bifurcations [35,36]. In Boutyour's work [33,37], the detection of a bifurcation point can also rely on the analysis of the roots of the denominator of the Padé approximants. In addition, Lopez [38] selected the expansion parameter based on the residual vector, and Mottaqi [39] discussed some other numerical techniques to detect the bifurcation point along the equilibrium path, based on Potier-Ferry's work [34].

The Koiter–Newton method [40–42] is a recently proposed reduced order modeling methodology to trace the entire equilibrium path in a stepwise manner. At each step, the approach combines a prediction using a non-linear reduced order model (ROM) based on Koiter's initial post-buckling expansion [43–46] with a Newton arc-length correction, thus allowing the algorithm to use fairly large step sizes. The possibility of some perturbation loads that may excite the secondary branches is taken into account to deal with bifurcation-point-type buckling. We have found that for an exactly perfect structure, i.e. a flat plat under axial compression, the proposed method can directly trace one of the bifurcation branches after the bifurcation point [42], with no need to introduce any physical imperfection. The effectiveness of the proposed method for the limit-point-type buckling has been proven in [40], and a force imperfection model has been introduced in [41] to show the ability of imperfection analyses. However, recently we noticed that if the structure bifurcates before the limit point the proposed method may pass by the bifurcation branch leading to an overestimated stability design. Hence, a bifurcation indicator should be introduced in the Koiter–Newton method to detect the bifurcation point and to allow tracing possible branches accurately.

The contribution of this paper distinguishes significantly from previous publications in the strategy followed to find the bifurcation branches. Previously, these were found by the introduction of perturbation loads, a strategy which may fail in the case that the bifurcation branches lie before the limit point. In this paper we overcome this shortcoming by using a bifurcation indicator based on the eigenanalysis of the tangent stiffness. Before reaching the first limit point the energy preserving properties of the system are indicated by a positive definite tangent stiffness matrix, a property which is lost with the appearance of instabilities in the structure signaled by negative eigenvalues. Here, the tangent is obtained from the reduced order model with, in general, less than 10 degrees of freedom. This fact allows the use of this bifurcation indicator in each iteration step during tracing the equilibrium path. In this respect, this method differs not only from the previous ones presented in [40–42] but also distinguishes from the classical arc-length method, which always fails to deal with the bifurcation-point-type buckling if no physical imperfection is introduced. Another point which characterises this paper is the use of the critical mode as the displacement perturbation at the bifurcation point to let the primary path change into the expected bifurcation branch. The proposed bifurcation indicator designed in the framework of the Koiter–Newton method is also different from the one proposed by Boutyour et al. [33] which is well adapted to the Asymptotic Numerical Method. Boutyour's bifurcation indicator introduces a fictitious perturbation force in the problem and requires the computation of a second series at each step. The indicator proposed in this paper is simpler with respect to implementation aspects and the computation only refers to a lower-order reduced order model.

The paper is organized as follows. The developed bifurcation indicator in the framework of the Koiter–Newton method for detecting bifurcation branches is presented in Section 2. In this section we also briefly revisit the fundamental aspects of the original Koiter–Newton method to allow a clear comprehension of the strategy and the mechanism of the proposed bifurcation indicator. Numerical examples which demonstrate the success of the method are provided in Section 3. We summarize the paper and draw conclusions in Section 4.

2. A bifurcation indicator proposed in the framework of the Koiter–Newton method

The Koiter–Newton method was proposed, similar to classical path following techniques, as a step by step procedure to trace the complete equilibrium path of a deforming structure. In each expansion step of the Koiter–Newton method three basic steps are involved: (1) construction of the reduced order model, (2) iterative solution of the reduced order model, (3) correction of the predicted load-displacement step produced by the reduced order