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Non-linear response of curved sandwich panels – extended high-order approach

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ABSTRACT

The geometrical non-linear behavior a curved sandwich panel with a stiff or compliant core when subjected to a pressure load using the Extended High-Order Sandwich Panel theory (EHSAPT), is presented. The formulation follows the EHSAPT procedure where the in-plane. i.e circumferential rigidity of the core is considered and the distribution of the displacements through the depth of the core are presumed. These displacement distributions are the closed-form solutions of the 2D governing equations of the curved core without circumferential rigidity that appear in the HSAPT curved sandwich panel model. The mathematical formulation includes the field equations along with the appropriate boundary and continuity conditions that take into account the high-order stress resultants in the core due to the presumed distributions. Finally a numerical study is conducted for a panel loaded by a distributed pressure at the upper face sheet. It reveals that the post-buckling response of a curved sandwich panels is associated with shallow to deep wrinkling deformations of the upper face sheet in the case of a simply-supported panel or a general non-linear pattern without wrinkles in the case of pinned supports with a short span. In both cases a stable post-buckling response is observed similar to that of a plate one.

1. Introduction

In aerospace, naval and transportations industries, where weight savings combined with high strength and stiffness properties are always required, the use of curved sandwich structures is increasing. In general, sandwich panels are made of two thin face sheets, metallic or laminated composites, bonded to a core that is often made of honeycomb, polymer foam or balsa wood. The core usually provides the shear resistance/stiffness to the sandwich structure in the radial direction, and a kind of elastic radial support to the face sheets that yields radial normal stresses. Polymer foams of low density or low strength honeycomb cores, which are flexible in the radial direction and very flexible relative to the rigidities of the face sheets, may affect the global response as well as the local one through changes of the core height and the core cross section plane which may take a nonlinear deformed pattern. In general, when using low strength foams or honeycomb of any material the in-plane rigidity of the core is neglected. However, when high density foam, solid foams or balsa wood are used, the circumferential (in-plane) rigidity of the core must be considered in addition to the shear and the radial rigidities. Also with heavy foams or solid materials the core undergoes displacements pattern that change the height of the core and distorts its cross section plane, denoted as high-order effects. The inclusion of this in-plane rigidity, based on presumed displacement patterns that are non-polynomial which yields non-conventional high-order stress resultants, for the non-linear response of curved sandwich panel is one of the major goals of the paper.

Analyses of flat sandwich panels that appear in the literature can be cataloged by two major categories. The first one assumes that the cores are an anti-plane type, i.e. very stiff in the vertical directions (incompressible) and with negligible in-plane rigidity in the longitudinal direction (metallic honeycomb), see for example the textbooks by Allen [1], Plantema [27] and Vinson [35]. Such panels may be modeled by computational models such as First or High order shear deformable approaches that replace the layered panel with an equivalent single layer, ESL, and assume that the core is incompressible. The second approach is denoted as layered approach and it is described through high-order models, see for example Frostig et al. [13] that solved the core fields in a closed-form or Carrera and Brischetto [7] that presumed the displacements fields of the core. In such models the overall response is a combination of the responses of the face sheets and the core through equilibrium and compatibility.

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Nomenclature

ESL Equivalent Single Layer
FOSDT First-Order Shear Deformable Theory
HSAPT High-Order Sandwich Panel Theory
EHSAPT Extended High-Order Sandwich Panel Theory
ODEs Ordinary Differential Equations
α Angle of curved panel
β_j $(j=t,b)$ section plane slope of face sheets
δ Variational operator
δ_d Dirac function
ε_{oj}, χ_j $(j=t,b)$ mid-plane strain and displacement curvature of face sheets
θ^*_{ej} $(j=t,b)$ angle location from left side of concentrated load at one of face sheets
η_j $(j=t,b)$ mathematical quantity
λ_{jk} $(j=t,b,k=u,w)$ Lagrange Multiplier at face–core interfaces in circumferential and radial directions
μ_{rsc}, μ_{src} Poisson ratio of core in circumferential and radial directions $_{+\text{of core}}$
σ_{ssj}, ε_{ssj} $(j=t,b,c)$ longitudinal normal stresses and strains in face sheets and core
σ_{rrc}, ε_{rrc} radial normal stresses and strains in core
τ_{rsc}, γ_{rsc} shear stresses and shear angle in core
τ_{cj},σ_{cj} $(j=t,b)$ shear and vertical normal stresses at upper and lower face–core interfaces
$\zeta=0,1$ boundary condition parameter
b_w width of panel
c thickness of core
d_j $(j=t,b)$ thickness of face sheets
e subscript that defines external loading magnitude and location
EA_j, EI_j $(j=t,b)$ in-plane (circumferential) and flexural rigidities of face sheets
E_{cr}, E_{cs},G_c modulus of elasticity in radial and circumferential directions and shear modulus of core
$f_k=\partial f/\partial k$ $(k=x,z)$ function derivative with respect to various variables
GA_c shear rigidity of panel
$j=t,b,c$ face sheets and core subscript indices
$D(w_j)(\phi)$ or Dw_j $(j=t,b)$ slope of radial displacement of face sheet
n_j, q_j $(j=t,b)$ External distributed loads in the circumferential and radial directions, respectively,
N_{ej}, P_{ej}, M_{ej} $(j=t,b)$ External concentrated loads in the circumferential and radial directions applied at the face sheets
N_{ssj}, M_{ssj}, V_{srj} $(j=t,b)$ in-plane (circumferential), bending moment and radial shear stress resultants of the face sheets
N_{ssc}, M_{ssc}, Q_{src} in-plane (circumferential), bending moment and radial shear stress resultants of the core
M_{ss2c}, M_{ss3c}, M_{Qsr1c}, M_{Qsr2c}, M_{Rrc}, R_{rc} High-Order stress resultants quantities in core
N_{ssej}, M_{ssej},V_{srej} $(j=t,b,c)$ Equivalent stress resultants: in-plane (circumferntial), bending moment, and radial shear of face sheets and core
r,θ radial and circumferential coordinates of panel
r_k $(k=t,b,ce,ct,cb)$ radius of centroid of upper face sheet, lower face sheet and core and radii of face–core interfaces respectively
U,U_λ,V potential energy of strain, constraints energy and external potential energy of loads
$u_{oj}(\theta,r_j), w_j(\theta,r_j), \beta_j(\theta,r_j)$ $(j=t,b)$ Mid-plane circumferential, radial and section plane slope the face sheets
u_c (θ,r_c), w_c (θ,r_c) circumferential and radial displacements of the core
$u_k(\theta,r_c)(k=o,1..3), w_k(\theta,r_c)(k=o,1..2)$ Displacement patterns in core of circumferential and radial displacements
y Coordinate in width direction
z_j $(j=t,b)$ radial coordinate of each face sheets
Comment Letters without indices refer to overall quantities

Research on curved or shell sandwich panel includes ESL computational models that consider an incompressible core following the First-Order Shear Deformable (FOSDT) Theory due to Reissner or Reissner–Mindlin works. This approach has become the basis for a large number of research works and to mention a few: Whitney and Pagano [36] and Noor et al. [24,25] for flat and curved panels respectively; Kollar [19] investigated buckling of generally anisotropic shallow sandwich shells and Vaswani et al. [34] performed vibration and damping analysis of curved sandwich beams. The last two models use Flügge shell theory while assuming that the face sheets are membranes only and the core is incompressible. Shallow cylindrical sandwich panels with orthotropic surfaces adopting the FOSDT models has been investigated by Ying-Jiang [37], but different from the others the face sheet here have membrane and flexural rigidities. Similarly using the principle of virtual work along with the FOSD model along with Sanders non-linear stress-displacement relations, Di Sciuva [9] and Di Sciuva and Carrera [10] have developed a model that take into account the shear deformation but assuming that the core is incompressible and linear. A stability analysis for cylindrical sandwich panels with laminated composite faces based on the Reissner hypothesis has been derived by Rao and Meyer-Piening [28,29] again with an incompressible core and membrane face sheets…

A class of high-order models based on the assumption of cubic and quadratic or trigonometric through-thickness distributions for the displacements have been suggested and summed in a review by Librescu and Hause [21], mainly on incompressible core. For cylindrical shells, with a compressible core, Hohe and Librescu [22] used a cubic and quadratic polynomial tangential and radial displacements distributions respectively while in Hohe and Librescu [23] a quadratic distribution has been used for the displacement distributions. All the referenced high-order models use integration through the thickness along with variational principle.

A different approach that includes the effect of the transverse (radial) normal stresses on the overall behavior of sandwich shells has been considered by Kuhhorn and Schoop [20]. They used geometrically non-linear kinematic relations along with pre-assumed polynomial deformation patterns for plates and the same for shells. The effect of the compressibility of the core using the HSAPT approach has been implemented in a number of research works on curved sandwich panel and to mention a few: Bozhevolnaya and Frostig [3] for non-linear behavior, Bozhevolnaya's Ph.D. thesis [4] on shallow sandwich panels, Karayadi's Ph.D. thesis [17] on cylindrical shells, Frostig [14] on the linear behavior of curved sandwich panels, Bozhevolnaya and Frostig [5] on the free vibration of curved panels, Thomsen and Vinson [33] on composite sandwich aircraft fuselage structures. Recently, thermal effects that include induced deformation as well as degradation of properties have been implemented in the analyses of curved sandwich panel, see Frostig and Thomsen [15] on fully bonded panels and Frostig and Thomsen [16] for panels with a