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The Riemann problem for non-ideal isentropic compressible two phase flows

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ABSTRACT

We consider the Riemann problem for a five-equation, two-pressure (5E2P) model of non-ideal isentropic compressible gas–liquid two-phase flows. This system is more complex due to the extended thermodynamics model for van der Waals gases, that is, typical real gases for gas phase and Tait's equation of state for liquid phase. The overall model is strictly hyperbolic and non-conservative form. We investigate the structure of Riemann problem and construct the solution for it. To construct solution of Riemann problem approximately assuming that all waves corresponding to the genuinely non-linear characteristic fields are rarefaction and then we discuss their properties. Lastly, we discuss numerical examples and study the solution influenced by the van der Waals excluded volume.

1. Introduction

Two-phase flows, and in general multiphase flows, are important in a large range of industrial applications, such as in the oil and gas industry, in the chemical and process industry, in the safety analysis of nuclear power plants as well as in a wide variety of scientific applications such as plasma physics, solid state physics, optical fibers, and fluid dynamics etc. Mathematical modeling of many physical systems lead to non-linear ordinary and partial differential equations. Generally, we do not have the luxury of complete exact solution for a non-conservative system of hyperbolic partial differential equations, for analytical work we have to rely on some approximate analytical techniques which may be useful to set the view and provide useful information towards our understanding of the complete physical phenomena involved.

There are several two-phase flow models in the literature [1–8]. Two-phase flows can be described by means of different models: mixture, drift (homogeneous or not), two-fluid or even multi-field models are currently used in industrial thermo-hydraulic codes. There is an important issue regarding these models that those are either conservative or non-conservative form, that is to say, whether the basic equations have, or not, a known conservation-law form in mathematical sense. In the absence of a conservative form of the equations one speaks of non-conservative model, even though in the derivation of the equations one invokes physical conservation principles. Since these models contain non-conservative products and therefore, Rankine–Hugoniot relations cannot be defined in an unambiguous manner. In particular, an additional closure relation is needed to provide a full set of jump conditions. For the solution of compressible two-fluid models, one can use exact Riemann solver or approximate Riemann solver. Andrianov and Warnecke [9] were the first to propose an inverse solution to the Riemann problem in the sense that the initial left and right states are obtained as a function of the intermediate states of the solution. Another direct approach to construct theoretical solution, proposed by Castro and Toro [10], of the Riemann problem for the five-equation two-phase non-conservative model of Saurel and Abgrall [4]. Murrone and Guillard [11] have studied an Eulerian diffuse interface model of Baer–Nunziato [12] type for the simulation of compressible multi-fluid and two-phase flow problems. A Riemann solver derived by a relaxation technique for classical single-phase shallow flow equations and for a two-phase shallow flow model describing a mixture of solid granular material and fluid is presented by Pelanti et al. [13]. A Robust and an accurate Riemann solver for isentropic drift-flux model of two phase flows have been analyzed by Kuila et al. [14]. Using the theory of progressive waves and some related procedures, waves of finite and moderately small amplitudes, influenced by the effects of non-linear convection, attenuation and geometrical spreading are studied by Ambika et al. [15] in an imperfect gas modeled by the van der Waals equation of state. Solution of the Riemann problem in magnetogasdynamics have

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been studied by Singh and Singh [16]. Chadha and Jena [17] used the Lie group of transformations and obtained the whole range of self-similar solutions to the problem of propagation of shock waves through a non-ideal dusty gas. Kuila et al. [18] presented computational simulations and analytical techniques for solving the drift-flux two-phase flow model under isothermal conditions.

In this work, a non-conservative five equation compressible two-phase model of Ransom and Hicks [3] is considered. The two-phases are made of namely the gas phase and the liquid phase. The mathematical model comprises 2-pressure of an isentropic equation of state for van der Waals gas and Tait's pure liquid. Since at high temperatures and low density the assumption that the gas is ideal and it is no longer valid, the popular alternative to the ideal gas is a simplified van der Waals model. Admissible shock waves and shock-induced phase transitions in a van der Waals fluid have been studied by Zhao et al. [19]. Ansari and Ghiasi [20] considered the model of [3] and investigated the hydrodynamical instability initiation criterion in two-phase air–water stratified flow using spectral method in a horizontal duct. The two-rarefaction approximation in single-phase gas dynamics was introduced by Toro [21]. We refer Sharma [22] for existence of weak solutions to the Riemann problem for the conservative hyperbolic systems. In our study, we extend this approach to solve the hyperbolic model, pioneered by [3], approximately assuming that all the genuinely non-linear characteristic fields are associated with rarefaction waves. If all genuinely non-linear waves are in fact rarefaction, then the obtained solution is exact. Otherwise, our solution will be an approximation. On this approximation, finally, we obtain a system of non-linear algebraic equations and we solve them using Newton–Raphson iterative procedure with a stopping criterion where the relative error is less than 10^{-8}; the initial guess for the intermediate unknown physical quantity is taken to be the average of left and right states.

The summary of this work is as follows. In the Section 2, the five equation two pressure model is proposed, and its conservative and primitive variable formations are analyzed. The Riemann problem and its characteristic framework are briefly discussed in Section 3. In Section 4, we assume that all genuinely non-linear characteristic fields are associated with rarefaction waves and discuss the properties of the rarefaction waves. In Section 5, we present the solution strategy of the Riemann problem and solvability of system of non-linear algebraic equations. Numerical tests to illustrate different choices are discussed briefly in Section 6. Section 7 is devoted to some concluding remarks and ideas for further future work.

2. Two phase flow model

We consider the two-phase flow model of Ransom and Hicks [3] that governs the dynamics of two compressible fluids (called phases, hereafter), namely the gas phase and the liquid phase.

2.1. Non-conservative form

The 5-equation, 2-pressure (5E2P) model which can be written in the non-conservative form as [20]:

$$\frac{\partial}{\partial t}(\alpha_1\rho_1)+\frac{\partial}{\partial x}(\alpha_1\rho_1 u_1)=0, \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_2\rho_2)+\frac{\partial}{\partial x}(\alpha_2\rho_2 u_2)=0, \tag{2}$$

$$\frac{\partial}{\partial t}(\alpha_1\rho_1 u_1)+\frac{\partial}{\partial x}(\alpha_1\rho_1 u_1^2)+\alpha_1\frac{\partial p_1}{\partial x}+(p_1-\tilde{p})\frac{\partial \alpha_1}{\partial x}=0, \tag{3}$$

$$\frac{\partial}{\partial t}(\alpha_2\rho_2 u_2)+\frac{\partial}{\partial x}(\alpha_2\rho_2 u_2^2)+\alpha_2\frac{\partial p_2}{\partial x}+(p_2-\tilde{p})\frac{\partial \alpha_2}{\partial x}=0, \tag{4}$$

$$\frac{\partial \alpha_1}{\partial t}+\tilde{u}\frac{\partial \alpha_1}{\partial x}=S. \tag{5}$$

Index 1 in the above system is referred to the liquid phase and 2 to the gas phase; x is the space coordinate and t is the time; ρ_i, u_i, p_i and α_i are the density, velocity, pressure and volume fraction of phase i $(i=1,2)$, respectively. The volume fractions are subject to the constraint

$$\alpha_1+\alpha_2=1, \tag{6}$$

and S is the source term on the right hand side of Eq. (5). Also, $\tilde{p}$ and $\tilde{u}$ denote the interface pressure and horizontal velocity which are respectively given by

$$\tilde{p}=(a_1p_2+a_2p_1)/(a_1+a_2) \tag{7}$$

and

$$\tilde{u}=(u_1+u_2)/2, \tag{8}$$

where $a_i=c_i\rho_i$ is the acoustic impedance and $c_i^2=dp_i/d\rho_i$ is the sound speed of material i $(i=1,2)$.

We use Tait's equation of state for liquid phase in the following form [10]:

$$p_1=p_1(\rho_1)=K_1\left[\left(\frac{\rho_1}{\rho_0}\right)^{\gamma_1}-1\right], \tag{9}$$

where K_1, γ_1 and ρ_0 are constants to be specified.

In particular, for the gas phase, we consider a van der Waals gas obeying the equation of state [23]:

$$p_2=p_2(\rho_2)=K_2\left(\frac{\rho_2}{1-b\rho_2}\right)^{\gamma_2}, \tag{10}$$

where K_2 is the constant, γ_2 is the specific heats ratio lying in the region $1<\gamma_2<2$ and b the van der Waals excluded volume, which lies in the range $0.9\times10^{-3}\le b\le 1.1\times10^{-3}$ and satisfying $1\gg b\rho_2\ge 0$. It may be noticed that the case $b=0$ corresponds to the ideal gas.

Therefore, the sound speeds for liquid and gas phases are

$$c_1=\sqrt{\frac{K_1\gamma_1\rho_1^{\gamma_1-1}}{\rho_0^{\gamma_1}}}\quad\text{and}\quad c_2=\sqrt{\frac{K_2\gamma_2\rho_2^{\gamma_2-1}}{(1-b\rho_2)^{\gamma_2+1}}}, \tag{11}$$

respectively.

Eqs. (1)–(5) form a first-order, quasi-linear and non-conservative system of partial differential equations for isentropic gas–liquid two-phase flow.

2.2. Conservative and primitive-variables formulation

To further establish the necessary mathematical framework of the current model equations for the development of a Riemann solver, we write the basic equations in the following form of conservative and primitive variables, respectively

$$U=[\alpha_2\rho_2,\alpha_2\rho_2u_2,\alpha_1\rho_1,\alpha_1\rho_1u_1,\alpha_1]^{tr}, \tag{12}$$

and

$$V=[\rho_2,u_2,\rho_1,u_1,\alpha_1]^{tr}, \tag{13}$$

where tr denotes the transposition. The two-phase model derived by [3] in the quasi-linear form with the conservative variable U as

$$\frac{\partial U}{\partial t}+M(U)\frac{\partial U}{\partial x}=0, \tag{14}$$