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Review

On the non-linear stability of a continuous duopoly model with constant conjectural variation

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ABSTRACT

The paper concerns a continuous model governed by a ODE system originated by a discrete duopoly model with bounded rationality, based on constant conjectural variation. The aim of this paper is to show (i) the existence of an absorbing set in the phase space; (ii) linear stability analysis of the critical points of the system; (iii) non-linear, global asymptotic stability of equilibrium of constant conjectural variation.
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1. Introduction and the model

The classic model of oligopoly was proposed by the French mathematician Cournot [9] and dates back to 1838. The oligopoly market structure showing the action of only two companies is called duopoly. Duopoly is an intermediate situation between monopoly and perfect competition, and analytically is a more complicated case, because an oligopolist must consider not only the behavior of the costumers, but also that one of the competitors and their reactions. In duopoly game each duopolist believes that he can calculate the quantity he should produce in order to maximize his profits. In the study of duopoly theoretic and realistic problems, the Cournot model and successively the conjectural variation model proposed by Bowley in 1924 [5] and later by Frish in 1933 [14] have been generally accepted and became the two important tools to describe the market behavior: conventional game theory and conjectural variation model. A useful summary of the history and of the debate on the conjectural variations is provided by Giocoli in [15]. Conjectural variation is referred to the beliefs that one firm has about the way its competitor may react if it varies its output. The firm forms a conjecture about the variation in the other firm's output that will accompany any change in its own output. In the classic Cournot model, it is assumed that each firm treats the output of the other firm as given when it chooses its output. In this case the conjectural variation is zero.

Many researchers have paid great attention to the dynamics of games (see for instance [1–4,11,16]). In fact, they tried to make theory more realistic, by presenting some various bounded

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rationality behaviors, combining game theory with dynamic systems and by introducing a delay in terms of bounded rationality. All these aspects gave birth to the bounded rational dynamic game. Several types of expectations have been proposed, among them are naive expectations, adaptive expectations and bounded rationality. In order to take into account the diffusion in the territory whose outputs are in the market, a generalization via a continuous PDEs system with the self and cross diffusion terms has been proposed by [25,22]. We recall that the reaction–diffusion systems of PDEs are the best candidates for investigating the diffusion processes in spatial domains (see for instance convection in porous media [7,8,10,20], biological systems [6,21], different non-linear diffusion problems [23,24,13,26–28] and references therein). In particular, when the outputs are in the market in large territories, a generalization for the Cournot–Kopel duopoly model via a continuous PDEs system [25,22], with self and cross diffusion, has been proposed.

Given a model, economists need to make predictions on the asymptotic behavior of the system i.e. how the model behaves in the future. Although the mathematical models represent only an approximation of the problem (they consider only some variables that are involved in the phenomenon), they allow to obtain an estimate for the market development.

Recently, a discrete duopoly model [29], with bounded rationality based on constant conjectural variation has been proposed. Precisely, it is assumed that two duopolists X and Y, in the hypothesis of constant conjectural variation, produce similar products for quantity competition. The firms do not have complete information or complete rationality behavior, but they know marginal profit function of constant conjectural variation. This is a model which utilizes marginal profit function of static constant conjectural variation to make output decision. The marginal profit function of conjectural variation of two firms X and Y has the following form

$$\begin{cases} \Pi_x(x,y) = \theta_1 - \gamma y - L_1 x \\ \Pi_y(x,y) = \theta_2 - \gamma x - L_2 y \end{cases} \tag{1}$$

with $\theta_i = \alpha_i - c_i > 0$ (i=1,2) where $c_i > 0$ is the marginal cost function, while α_i, L_i and γ are positive model parameters. Denoted by x_t and y_t the outputs of firms X and Y, respectively, the firms adjust the output of the next period $t+1$ considering marginal profit function and output of the current period t. When marginal profit function is positive the output is increased otherwise it is decreased. One obtains the following discrete system for the two firms

$$\begin{cases} x_{t+1} = x_t + a x_t(\theta_1 - \gamma y_t - L_1 x_t) \\ y_{t+1} = y_t + \nu y_t(\theta_2 - \gamma x_t - L_2 y_t) \end{cases} \tag{2}$$

where $a > 0$ and $\nu > 0$ represent the speeds of output adjustment of two firms. In [29], the local stability of critical points, numerical simulations, Liapunov exponents and fractal dimensions of strange attractors are investigated. In the present paper, we generalize (2) via a continuous ODEs system. Assuming continuous time scales, denoted by u and v the outputs of the two firms X and Y, respectively, from (2) the non-linear continuous system for the evolution of u and v is immediately obtained

$$\begin{cases} \dfrac{du}{dt} = au(\theta_1 - \gamma v - L_1 u) \\ \dfrac{dv}{dt} = \nu v(\theta_2 - \gamma u - L_2 v) \end{cases} \tag{3}$$

where

$$\phi : t \in R^+ \to \phi(t) \in R, \quad \phi \in \{u, v\}.$$

To (3) we append the initial data

$$u(0) = u_0, \quad v(0) = v_0 \tag{4}$$

with u_0, v_0 being assigned positive constants.

The aim of the paper is to show

(i) the existence of an absorbing set of the solutions in the phase plane;
(ii) linear stability analysis of the critical points of the system;
(iii) non-linear, global asymptotic stability of the equilibrium of constant conjectural variation.

The paper is organized as follows. Section 2 is devoted to the existence of an absorbing set and uniqueness of the solution. Critical points of the system are determined in Section 3, linear stability of equilibria is performed in Section 4 while non-linear global asymptotic stability of the equilibrium of constant conjectural variation is shown in Section 5. Some final comments are collected in Section 6. The paper ends with an Appendix (Section 7) in which the origin of the function (44) is recalled.

2. Ultimately boundedness (absorbing sets) and uniqueness of the solution

We remark that the first orthant is invariant. In fact, integrating (3), one obtains

$$\begin{cases} u = u_0 \exp \int_0^t a(\theta_1 - \gamma v - L_1 u)\, d\tau \\ v = v_0 \exp \int_0^t \nu(\theta_2 - \gamma u - L_2 v)\, d\tau \end{cases} \tag{5}$$

and hence $\{u_0 > 0, v_0 > 0\}$ imply $\{u(t) > 0, v(t) > 0 \ \forall t \ge 0\}$ i.e. the first orthant is invariant.

As is well-known a set $\mathcal{A}$ of the phase space (u,v), is an absorbing set if, denoted by $d[(u,v),\mathcal{A}]$ the distance between $[u(t), v(t)]$ and $\mathcal{A}$ at time t, i.e.

$$d(t) = \inf_{\mathcal{A}} (|u - \overline{U}|^2 + |v - \overline{V}|^2), \quad (\overline{U}, \overline{V}) \in \mathcal{A}, \tag{6}$$

it follows that

(i) $\mathcal{A}$ is a global attractor, i.e.

$$\lim_{t\to\infty} d[(u,v),\mathcal{A}] = 0, \tag{7}$$

with $(u_0, v_0) \in \mathcal{B}$, for any open set $\mathcal{B} \supset \mathcal{A}$;

ii) $\mathcal{A}$ is positively invariant, i.e.

$$(u_0, v_0) \in \mathcal{A} \Rightarrow (u(t), v(t)) \in \mathcal{A}, \quad \forall t > 0. \tag{8}$$

Theorem 1. *Any set containing the rectangle*

$$S = \left\{ (u,v) \in R_+^2 : 0 < u \le \frac{\theta_1}{L_1}, \quad 0 < v \le \frac{\theta_2}{L_2} \right\}, \tag{9}$$

of the phase space is an absorbing set for (3).

Proof. By virtue of $(3)_1$ it follows that

$$\frac{du}{dt} = au(\theta_1 - \gamma v - L_1 u) \le au(\theta_1 - L_1 u) \tag{10}$$

from which, by setting $w = \dfrac{1}{u}$, we obtain

$$\frac{dw}{dt} \ge -a\theta_1 w + aL_1, \tag{11}$$

and hence, integrating, it follows that

$$w \ge w_0\, e^{-a\theta_1 t} + \frac{L_1}{\theta_1}(1 - e^{-a\theta_1 t}). \tag{12}$$

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