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Periodically forced system with symmetric motion limiting constraints: Dynamic characteristics and equivalent electronic circuit realization

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ABSTRACT

A two-degree-of-freedom periodically-forced system with symmetric motion limiting constraints is considered in this paper. The incidence relation between dynamics and constraint parameters (clearance and constraint stiffness) is studied and some novel results are obtained by double-parameter simulation analysis. The fundamental group of impact motions having the excitation period and differing by the number of impacts is given special consideration for analyzing low frequency vibration characteristics of the system. Dynamics of the system are studied with emphasis on the mutual transition characteristics between neighboring regions of fundamental impact motions. An electronic circuit is designed for physical implementation of dynamics of the periodically-forced system with symmetric constraints. The non-linear terms of the system are replaced by using an absolute-value function and can be fully implemented with simple electronic elements (resistors and operational amplifiers). The electronic circuit is realized and studied. The oscilloscope outputs of electrical waveforms of various non-smooth oscillations, generated by the circuit itself, are experimentally observed. A good agreement among the numerical results of the mechanical model, the electronic design simulation of the circuit and the real oscilloscope outputs of hardware implementation is confirmed.

1. Introduction

Dynamics of non-smooth mechanical systems, caused by piecewise smooth mechanical factors including clearance, constraint, dry friction and hysteresis, etc., has been an important research subject drawing great interest in the mechanical engineering field during the past years. The general characteristics for most of such systems are that their motion trajectories are smooth or even linear before the trajectories begin to contact the switching boundaries. However, on the switching boundaries something that causes non-linearities or even non-smooth properties occurs by its very nature due to impact, backlash, free-play, switching, dry friction or sticking, etc. Most crucially, the existence of the clearances or constraints brings about a possibility of impact occurrence between the constraints and parts of the mechanical systems and the impacts can, more or less, increase noise levels and intensify the abrasion of related parts. It is well known that such systems can exhibit not only all the non-linear phenomena that smooth dynamical systems can undergo, such as saddle-node bifurcation, Hopf bifurcation, homoclinic bifurcation, period doubling bifurcation and other non-local ones, etc. [1–15], but also display many of non-standard dynamical features caused by discontinuity-induced bifurcations that are unique to piecewise-smooth systems. Complex dynamics of non-smooth systems has been found to result from the occurrence of the grazing singularity of mappings, and the relevant theories of grazing bifurcation have been unceasingly deepened as seen in Refs. [13–31], with the pioneering work due to Nordmark [16]. Besides grazing bifurcation, richness and complexity in non-linear dynamical behavior induced by the non-smoothness, such as sticking [32], sliding [33,34], border-collision [35] and chattering [36–45] etc., have also been reported.

The experimental methods, based on practical systems or mechanical models, have been developed for testing and verifying dynamic characteristics of some non-smooth periodically-forced systems, as partly reported in Refs. [46–55]. Stensson and Nordmark [46] investigated the effects of low-velocity impacts through experimental and numerical efforts. Wiercigroch and Sin [47] performed the experimental study of base excited symmetrically piecewise linear oscillator. Jin and Hu [48] designed a vibro-impact experimental apparatus consisting of two cantilever beams with lumped masses and tested various vibro-impact motions. Nguyen, Woo and Pavlovskaia [49] designed an experimental scheme of vibro-impact rig to mimic a

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mole penetration through the soil. The experimental apparatus mainly consists of a metal bar placed within a solenoid connected to an RLC circuit and an obstacle block positioned near the bar. Subsequently, Ing, Pavlovskaia and Wiercigroch et al. [50] accomplished the experimental study of an impact oscillator with one sided elastic constraint. The experimental investigation and the mathematical modeling of the impact force behavior in a vibro-impact system were done by Aguiar and Weber [51], and the experimental data were used to validate the mathematical model. Wen and Xu [52] designed a vibro-impact experimental device consisting of an obstacle with adjustable gap sizes and two mass blocks linked by spring-damper elements, driven by the harmonic exciting forces and moving on a lead rail. Important advances in experimental vibro-impact dynamics of elastic structures have been obtained in recent years. Balachandran [53] studied the dynamics of a thin-walled structure subjected to impact excitations with the help of experimental and numerical investigations. Period-doubled motions, incomplete period-doubling sequences, aperiodic motions, and multiple responses of the elastic structure were observed during harmonic impactor motions. The numerical results show many qualitative similarities with the experimental results. Long et al. [25] studied non-smooth dynamics of an elastic structure excited by a harmonic impactor motion through a combination of experimental, numerical and analytical efforts. Soft impact between the impactor and the structure is considered, and qualitative changes that can be associated with grazing and corner-collision bifurcations are experimentally observed in Ref. [25]. Further, Long et al. [54] expanded the study to two harmonically excited elastic structures with impact interaction. Through associated experiments, typical behaviors of the non-smooth system, including grazing bifurcation, jump and hysteresis, are examined, which are in good agreement with numerical results. Although analytical and numerical results on grazing impacts and post-grazing phenomena have been reported extensively, experimental evidence of grazing impacts and post-grazing phenomena in mechanical and structural systems as reported in Refs. [25,46,50,53,54] has been limited.

Actually, an electronic circuit, designed starting from the mechanical model, can be effectively implemented and its dynamical behavior can be generalized observing the oscilloscope output of electrical waveforms generated by the circuit itself. However, we note that most studies of physical implementation of electric circuits describing non-linear dynamics have been based on canonical Chua' circuit [56–58], improved canonical Chua's circuit [59–61] and smooth non-linear systems (Lorenz equations and coupled Duffing oscillators) [62–66] in the past several years. Very few people have considered the electronic analog of non-smooth mechanical systems. Zimmerman et al. [67] realized the electronic analog of the bouncing ball by using a precision diode rectifier circuit, and the implemented experiment illustrated effectively some important features of the non-linear system. Clark et al. [68] addressed the problem of distinguishing regular, chaotic, and random behavior using an electronic circuit modeling of a ball bouncing on an oscillating table. A mechanical impact oscillator and a diode rectifier circuit for physical implementation of the oscillator were presented and their time series and phase portrait analysis were performed in Ref. [69]. Srinivasan et al. [70] studied dynamics of Duffing oscillator subjected to different non-sinusoidal periodic forces like square wave, triangle wave, sawtooth wave via the implementation of an electronic circuit modeling the Duffing equation. Most of numerical studies are in good agreement with observations from analog circuit experiments. Ho and Nguyen [71] studied non-linear dynamics of a new electro-vibro-impact system and put forward an implementation scheme of circuit analog.

In the present paper we study dynamics of a periodically-forced system with bilateral elastic constraints and concurrently emphasize the design and hardware implementation of electronic circuit describing dynamic characteristics of the non-smooth system. The paper is organized as follows. In Section 2, the mechanical model of a two-degree-of-freedom periodically-forced system with symmetric motion limiting constraints is introduced, and an appropriate rescaling of parameters and variables of the system is presented for studying the influence of the constraint parameters on diversity and evolutionary characters of periodic-impact motions in relatively large parameter spaces. In Section 3, the incidence relation between dynamics and non-linear factors (clearance and constraint stiffness) is studied by double-parameter simulation analysis. In Section 4, an electronic circuit is designed and made by using PCB for physical implementation of dynamics of the periodically-forced system with motion limiting constraints. In Section 5, partial numerical results of the mechanical model are experimentally verified by the signal test and analysis of the PCB. Finally, concluding remarks are given in the last section.

2. Mechanical models and dynamic equations

A two-degree-of-freedom periodically-forced system with motion limiting constraints is shown schematically in Fig. 1. The mass blocks M_1 and M_2 linked by linear spring-damper elements K_i and $\bar{C}_i$ ($i=1$, 2) are driven by the harmonic exciting forces with the amplitudes P_1 and P_2, exciting frequency Ω and phase angle τ. The displacements of two mass blocks are represented by X_1 and X_2, respectively. The stiffness of the constraint springs is labeled by K_0. Two cases, associated with the normal clearance and the prepressing constraints, are considered in the following study. The normal clearance signifies that right and left constraints are symmetrically situated in certain distances B and $-B$ from the equilibrium position of the mass block M_1, respectively. As shown in Fig. 1(a), two constraints are non-prepressing and a gap between the constraints and the mass block M_1 is retained. The prepressing constraints mean that the right (left) elastic constraint with a precompressed spring length B ($-B$) is symmetrically situated in the equilibrium position of the mass block M_1, as seen in Fig. 1(b). In this case these two constraints symmetrically exert the preloads $-K_0B$ and K_0B on the mass block M_1 due to the prepressing effect of the constraint springs attached to the supporting base. As $|X_1|=B$, the mass block M_1 begins to contact the constraint or break away from it, which hinges on the motion direction of the mass block M_1 at the

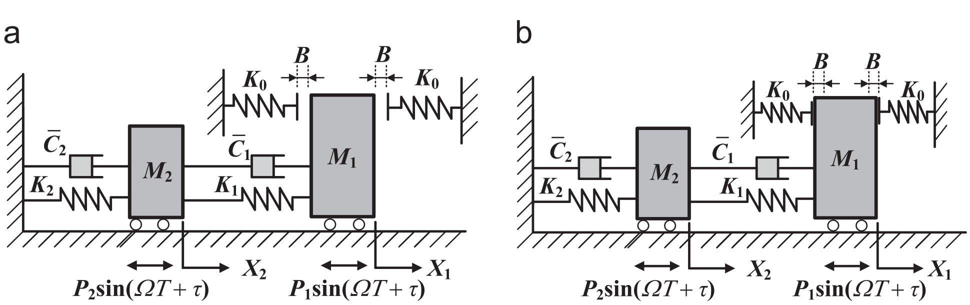

Fig. 1. Schematics of periodically-forced systems with motion limiting constraints represented by the unprestressed or prestressed springs attached to the supporting base: (a) unprestressed springs; (b) prestressed springs.