International Journal of Mechanical Sciences 93 (2015) 1–7

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Vibration and stability of a double-beam system interconnected by an elastic foundation under conservative and nonconservative axial forces

Qibo Mao [a,*], Nuttawit Wattanasakulpong [b]

[a] School of Aircraft Engineering, Nanchang HangKong University, 696 South Fenghe Avenue, Nanchang CN-330063, PR China
[b] Department of Mechanical Engineering, Mahanakorn University of Technology, Nongchok, Bangkok 10530, Thailand

ARTICLE INFO

Article history:
Received 30 June 2014
Received in revised form
26 November 2014
Accepted 13 December 2014
Available online 13 January 2015

Keywords:
Adomian modified decomposition method
Double-beam system
Flutter instability
Divergence instability

ABSTRACT

The Adomian modified decomposition method (AMDM) is employed in this study to investigate the free vibration and stability of a cantilever double-beam system, which is continuously joined by a Winkler-type elastic layer. The free end of each beam is restrained by a translational spring and subjected to a combination of compressive axial and follower loads. Based on the AMDM, the governing differential equations for the double-beam system are represented as a recursive algebraic equation. By using boundary condition equations, the natural frequencies and corresponding mode shapes can be easily obtained simultaneously. The double-beam system becomes unstable in the form of either divergence or flutter with the increasing loads. Then the critical loads are discussed under different boundary conditions and the nonconservative parameters. Furthermore, the effect of the value of the spring stiffness on the critical loads for either flutter or divergence instability of the double-beam systems is studied.

1. Introduction

The double-beam system made of two parallel beams continuously joined by a linear Winkler-type elastic layer is widely used in various engineering fields, such as double-walled carbon nanotubes [1–6], tall building [7] and aeronautical engineering applications [8–10]. Different numerical or analysis methods such as the differential quadrature method [1,2,11], nonlocal elasticity theory [3,4], Galerkin-type state-space method [7], spectral element analysis [8] and classical differential equation method [12] have been used in solving free vibration problems of such structures. No attempt will be made here to present a bibliographical account of previous work in the area of the free vibration analysis of double-beam systems. A few selective recent papers are quoted [1–4,7,8,11,12] which provide further references on the subject.

However, a relatively few papers have been published on the stability analysis for double-beam systems. Zhang et al. [9,10] investigated the free transverse vibration and buckling (divergence) of a simply-supported double-beam system under compressive axial loading. And the explicit expressions for the natural frequencies have been derived, then the analytical solutions of the critical buckling loads are obtained. The calculation results show that the critical buckling loads of the double-beam systems depend on the axial force ratio of the two beams and the stiffness of the Winkler elastic layer. Stojanovic et al. [13,14] extended the work of Ref. [9] to analyze the forced vibration and buckling of a Rayleigh and Timoshenko double-beam system under axial loading. Similar to Refs. [9,10], only the simply supported boundary condition for double-beam system is considered in Refs. [13,14]. Kozic et al. [15] discussed the free vibration and buckling of a double-beam system continuously joined by a Kerr-type three-parameter layer under axial loading. The analytical solution for the critical buckling load of the system is derived based on the classical Bernoulli–Fourier method. The explicit expressions are presented for natural frequencies and the associated amplitude ratios of the two beams. Murmu and Adhikari [5] studied the axial instability problem of double nanobeam systems using nonlocal elasticity theory. Scale-effects in the in-phase and the out-of-phase buckling phenomenon are presented. Murmu et al. [6] further extended the nonlocal elasticity theory to analyze the buckling of the double nanoplate system under biaxial compression.

Until now, most of these studies have been done within the scope of the classical simply-supported boundary condition with negligible follower loads, it means that only the divergence behavior of the double-beam systems has been investigated. In this paper, a relatively new computed approach called AMDM [16–22] is used to analyze the flutter and divergence instability for a cantilever double-beam system. It is assumed that the free ends of both beams under consideration is restrained by translational springs and subjected to the combination of compressive axial and follower loads. AMDM is a powerful method for solving linear and nonlinear differential equations. The solution by using AMDM is considered as a sum of an infinite series, and rapid convergence to an accurate solution [16,17].

* Corresponding author. Tel.: +86 791 645 3569.
E-mail address: qibo_mao@yahoo.com (Q. Mao).

http://dx.doi.org/10.1016/j.ijmecsci.2014.12.019
0020-7403/

This paper is organized as follows. First, the governing differential equations for the double-beam systems are summarized in a matrix form. By imposing the AMDM, the governing differential equations in matrix form become recursive algebraic equations. And the boundary conditions become simple algebraic frequency equations which are suitable for symbolic computation. Then the natural frequency and corresponding closed-form series solution of mode shape can be obtained simultaneously. Finally, some numerical examples are presented. The influence of the ratio between the axial and follower loads on natural frequencies of the double-beam systems is discussed. Particular attention is devoted to the evaluation of the influence of the boundary conditions on the critical loads level for either flutter or buckling instability.

2. AMDM for the double-beam systems

Consider the free vibration of two elastically connected parallel cantilever beams, the free end is restrained by translational springs and subjected to combination of compressive axial and follower loads, as shown in Fig. 1. Both beams have the same length L. The total concentrated load on the nth beam is the sum of axial load $(1-\alpha_n)P_n$ and follower load $\alpha_n P_n$ $(n=1,\ 2)$. The parameter α_n is termed as nonconservative parameter [23]. The stability for qualitatively different loads can be investigated by varying the parameter α_n. For examples, $\alpha_n=0$ describes the pure compressive axial load on the nth beam, while the pure follower load is applied when $\alpha_n=1$. The values of $0<\alpha_n<1$ describe cases in which both axial (conservative) and follower compressive loads are applied simultaneously.

The Bernoulli–Euler model for transverse vibration is used in this study. Notice that the Bernoulli–Euler model assumes that both the rotary inertia and shear deformation are negligible. The ratios of the depth to the length of the beams in this study are assumed small. And the plane sections of the beams are assumed to remain plain and the curvatures of the beams are assumed to be small.

According to Ref. [9], the partial differential equation describing the free vibration in each beam, are derived using Euler–Bernoulli theory and can be expressed as follows:

$$E_1 I_1 \frac{\partial^4 w_1(x,t)}{\partial x^4} + \rho_1 A_1 \frac{\partial^2 w_1(x,t)}{\partial t^2} + P_1 \frac{\partial^2 w_1(x,t)}{\partial x^2} + k_s[w_1(x,t) - w_2(x,t)] = 0 \tag{1}$$

$$E_2 I_2 \frac{\partial^4 w_2(x,t)}{\partial x^4} + \rho_2 A_2 \frac{\partial^2 w_2(x,t)}{\partial t^2} + P_2 \frac{\partial^2 w_2(x,t)}{\partial x^2} + k_s[w_2(x,t) - w_1(x,t)] = 0 \tag{2}$$

where the transverse displacement of the nth beam is denoted as $w_n(x,\ t)$. E_n and ρ_n are Young's modulus and density of the nth beam, respectively. I_n is the cross-sectional moment of inertia of the nth beam, $I_n = (b_n h_n^3/12)$, $A_n = b_n h_n$ is the cross-section area. b_n and h_n are the width and thickness of the nth beam, respectively. k_s is the stiffness of the Winkler-type elastic layer between the beams. P_n is the total concentrated load which is positive in compression.

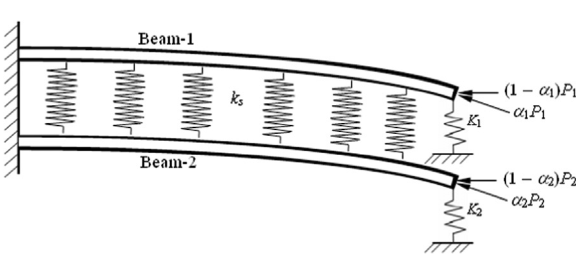

Fig. 1. Double-beam system under linear combination of compressive axial and follower loads.

According to modal analysis approach (for harmonic free vibration), the $w_n(x,\ t)$ can be separable in space and time:

$$w_n(x,t) = W_n^*(x)\, e^{i\omega t} \tag{3}$$

where ω and $W_n^*(x)$ are the natural frequency and the spatially dependent structural mode shape of the nth beam, respectively. $i=\sqrt{-1}$.

Substituting Eq. (3) into Eqs. (1) and (2), then separating variable for time t and space x, and introducing non-dimensional variable $X=(x/L)$ and $W_n(X) = (W_n^*(x)/L)$, the ordinary differential equation for each beam can be obtained:

$$\frac{E_1 I_1}{L^4}\frac{\partial^4 W_1(X)}{\partial X^4} = \omega^2 \rho_1 A_1 W_1(X) - \frac{P_1}{L^2}\frac{\partial^2 W_1(X)}{\partial X^2} - k_s[W_1(X) - W_2(X)] \tag{4a}$$

$$\frac{E_2 I_2}{L^4}\frac{\partial^4 W_2(X)}{\partial X^4} = \omega^2 \rho_2 A_2 W_2(X) - \frac{P_2}{L^2}\frac{\partial^2 W_2(X)}{\partial X^2} - k_s[W_2(X) - W_1(X)] \tag{4b}$$

Eqs. (4a) and (4b) can be simplified as a matrix form

$$\frac{d^4 \boldsymbol{W}(X)}{dX^4} = \boldsymbol{P}\frac{d^2 \boldsymbol{W}(X)}{dX^2} + \boldsymbol{T}_\omega\, \boldsymbol{W}(X) \tag{5}$$

where

$$\boldsymbol{W}(X) = \begin{bmatrix} W_1(X) \\ W_2(X) \end{bmatrix}, \boldsymbol{P} = \begin{bmatrix} \frac{L^2}{E_1 I_1} P_1 \\ \frac{L^2}{E_2 I_2} P_2 \end{bmatrix}, \boldsymbol{T}_\omega = \begin{bmatrix} \frac{L^4(\omega^2 \rho_1 A_1 - k_s)}{E_1 I_1} & \frac{L^4 k_s}{E_1 I_1} \\ \frac{L^4 k_s}{E_2 I_2} & \frac{L^4(\omega^2 \rho_2 A_2 - k_s)}{E_2 I_2} \end{bmatrix}$$

According to the principle of AMDM [16–22], $W(X)$ in Eqs. (4a) and (4b) can be expressed as in terms of an infinite series of convergent series

$$\boldsymbol{W}(X) = \begin{bmatrix} W_1(X) \\ W_2(X) \end{bmatrix} = \begin{bmatrix} \sum_{m=0}^{\infty} C_{1,m} X^m \\ \sum_{m=0}^{\infty} C_{2,m} X^m \end{bmatrix} = \sum_{m=0}^{\infty} \boldsymbol{C}_m X^m \tag{6}$$

where the unknown coefficient vector $\boldsymbol{C}_m$ will be determined recurrently.

Impose a linear operator $G=(d^4/dX^4)$, then the inverse operator of G is therefore a 4-fold integral operator defined by the following equation:

$$G^{-1} = \int_0^x \int_0^x \int_0^x \int_0^x (\ldots)\, dX dX dX dX \tag{7}$$

and

$$G^{-1} G[\boldsymbol{W}(X)] = ''(X) - \boldsymbol{C}_0 - \boldsymbol{C}_1 X - \boldsymbol{C}_2 X^2 - \boldsymbol{C}_3 X^3 \tag{8}$$

Applying Eqs. (4a) and (4b) with G^{-1}, we get

$$G^{-1} G[\boldsymbol{W}(X)] = G^{-1}\left[\boldsymbol{P}\frac{d^2 \boldsymbol{W}(X)}{dX^2} + \boldsymbol{T}_\omega\, \boldsymbol{W}(X)\right] \tag{9}$$

Comparison Eqs. (8) and (9), we get

$$\boldsymbol{W}(X) = \sum_{m=0}^{3} \boldsymbol{C}_m X^m + G^{-1}\left[\boldsymbol{P}\frac{d^2 \boldsymbol{W}(X)}{dX^2} + \boldsymbol{T}_\omega\, \boldsymbol{W}(X)\right] \tag{10}$$

Substituting Eq. (6) into the right hand of Eq. (10), we get

$$\boldsymbol{W}(X) = \sum_{m=0}^{3} \boldsymbol{C}_m X^m + \sum_{m=0}^{\infty} \frac{\boldsymbol{P}\boldsymbol{C}_{m+2}}{(m+3)(m+4)} X^{m+4} + \sum_{m=0}^{\infty} \frac{\boldsymbol{T}_\omega \boldsymbol{C}_m}{(m+1)(m+2)(m+3)(m+4)} X^{m+4} \tag{11}$$

Comparing Eq. (11) to Eq. (6), it is observed that the coefficient vectors $\boldsymbol{C}_m$ in Eq. (11) can be determined by using the following