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Computational homogenisation of periodic cellular materials: Application to structural modelling

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ABSTRACT

The present paper aims at investigating the homogenisation of cellular materials in view of the modelling of large but finite cellular structures. Indeed, computation costs associated with the complete modelling of such structures can be rapidly prohibitive if industrial applications are considered. The use of a homogeneous equivalent medium (HEM) for these cellular materials can be an efficient approach to address this issue, but it requires the calibration of relevant homogeneous equivalent laws (HELs). Here, the considered cellular materials are tube stackings. Various uni-axial and multi-axial loading cases have been simulated, through the finite element method, on representative volume elements of such periodic stackings. From these simulations, anisotropic compressible elasto-plastic constitutive equations have been identified for the HEL. The anisotropy of the yield surfaces is discussed depending on the pattern of the tube stacking (e.g. square or hexagonal). A validation of the identified laws is proposed by simulating uni-axial compression and simple shear tests on sandwich structures made of tube stackings for their cores. A systematic comparison, between the results obtained from the fully meshed structures and those obtained from the structures whose core has been replaced with its HEM, allows us to address the limitations of the HEM-based approach and the boundary layer effects observed on finite structures.

1. Introduction

Cellular materials have been widely studied for their various functionalities [1] which make them attractive for numerous applications, for instance in which impact resistance or acoustic absorption is required. From a mechanical point of view, high specific properties relative to the bulk are expected which could be very useful in the development of lightweight aeronautical frames. However, the modelling of such cellular materials presents some difficulties because of the issue of the separation of their different characteristic scales; the size of the constitutive cells is often in the same order of magnitude as the one of the structure. A refined modelling of the mechanical behaviour of these materials is thus necessary to simulate large structures (with many constitutive cells but without increasing the computation costs), but keeping information on the local mechanisms which govern their mechanical behaviour (such as localised plasticity in the cells). To address this issue, a modelling approach based on the identification and the use of a homogeneous equivalent medium is investigated here.

By using a multi-scale description of cellular materials, many authors were interested in the influence of the intrinsic morphological and material parameters of these materials on their effective mechanical properties. For instance, one can cite the works of Silva et al. [2,3], Fazekas et al. [4], Sanders and Gibson [5,6] and Marcadon and Kruch [7] concerning the effect of the architecture or those of Amsterdam et al. [8,9] Mangipudi et al. [10] and Marcadon et al. [11,12] addressing the effect of the constitutive material. Numerous studies investigated also the characterisation of the damage mechanisms which govern the failure of cellular structures, especially the collapse of their constitutive cells [13–16]. The scale transition between both the microscopic and the macroscopic scales allowed authors to study the initial macroscopic plastic yield stress from the first plastic hinge at the microscopic scale, such as proposed by Gasser et al. [17]. Alkhader and Vural [18] studied the beginning of plasticity in foams depending on the topology and the resulting dominant deformation modes. The macroscopic yield and the behaviour of cellular structures in their inelastic domain have been studied too. Experimental campaigns of characterisation have been carried out on various geometries under both quasi-static [19–21] and dynamic [22,23] loads, but also computational characterisations. Full scale models have been developed for various architectures of cellular materials. Depending on the geometry of the cells, authors proposed different models such as beam models at finite strains for Papka and Kyriakides [24], shell models of real geometries coming from tomography analyses for Caty et al. [25], or solid models in 2D or 3D for Marcadon et al. [11,21]. In order to get macroscopic mechanical responses, authors used homogenisation techniques too

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for non-periodic media, see for instance the work of Ostoja-Starzewski [26].Beam models were used to study the influence of the morphology, i.e. variations of the cell size or geometrical defects and their dispersion, on the elastic properties and the onset of plasticity [27–29]. More complex beam elements were also considered in advanced models in order to better capture the architecture and the local collapse mechanisms [10,30,31].

Cellular materials are increasingly used in large structures. Thus their behaviour under arbitrary load depending on the application has to be known in order to predict the behaviour of whole structures. One can either consider a full-scale model describing all the characteristic scales of the material, but the computation costs become rapidly too expensive. On the contrary, the heterogeneous architecture can be replaced favourably by a homogeneous equivalent medium (HEM) into the modelling [32,33]. However, this technique requires the accurate characterisation of the relation between the macroscopic stresses and strains. It can either be the result of nested finite element simulations and be used in a structural model, or it can be an effective material law. The FE^2 (nested finite element) scheme [34–36] has the advantage to need no assumption on the macroscopic mechanical response, but it remains costly when applied to large 3D structures. On the contrary, the HEM-based modelling is very efficient in terms of computation costs, but it results in a loss of information concerning the architecture of the cellular material and it must be rigorously identified to be relevant. Various macroscopic material models that fit the macroscopic behaviour of cellular materials have been proposed to take into account their compressibility and their anisotropy. They can be identified either from experimental results [37–39] or from periodic simulations as proposed by Tsuda et al. [40]. Depending on their base material properties and their architecture, the homogenised behaviour can be isotropic (mainly for random architectures) or can exhibit an anisotropy if a certain regularity in the organisation or a particular shape of the cells exist. For instance, a quadratic yield criterion has been proposed by Hill [41]. Similarly, several yield criteria were proposed by Karafillis and Boyce [42], Barlat et al. [43] or Bron and Besson [44] to model multi-axial non-quadradic yield flows. Moreover, in the particular case of cellular materials, the void inside or within the constitutive cells leads to the compressibility of the effective behaviour. A combination of both a shear criterion and a trace dependent criterion governing the compressibility was originally proposed by Green [45]. An isotropic compressible criterion was also proposed for metallic foams by Deshpande and Fleck [39], whereas generalisations of Green's quadratic model [45] were introduced by Badiche et al. [46] and Xue and Hutchinson [32] for anisotropic compressible behaviours. As an illustration of HEM-based approaches, Tsuda et al. [40] characterised a plate-fin structure. The multi-axial inelastic behaviour was characterised by simulating mechanical loads controlled in macroscopic strain up to 0.5%. The anisotropy, the compressibility and the strain rate dependence of the structure effective behaviour were studied and then a quadratic homogenised model was identified in both the elastic and the inelastic domains.

The present work aims at investigating the homogenisation of cellular materials too. Compared with the works of the literature aforementioned, higher strain levels are investigated to discuss the quadratic shape or not of the yield surfaces, depending on cellular architecture and stress concentration, and their evolution according to the level of cumulated plastic strain. Focus is also on the use of such HEM in structural modelling and on boundary layer effects. Reference is made to sandwich structures with a cellular core made of stacked tubes as 'model cellular structures'. The tubes are stacked following either a square pattern or a hexagonal one. First, in Section 2 the multi-axial behaviour of the cellular core is characterised through periodic homogenisation and by simulating various multi-axial loading cases on the unit cells of the two different arrangements of tubes considered. The simulations are performed up to 5% of macroscopic strain and a multi-axial representation of the plastic flow surfaces is proposed in the eigen-stress space from large number of finite element simulations. Then, Section 3 is dedicated to the identification of a HEM for each stacking type considered, square or hexagonal. To finish, in Section 4 the HEM identified previously is used for the modelling of finite sandwich structures made of a tube stacking core. A systematic comparison, between the results obtained form the reference calculations on the fully meshed structures (full-scale modelling) and those predicted by replacing the cores of the sandwich structures with their HEM, is proposed with the purpose of evaluating the relevance of the homogenised constitutive equations identified. A parametric study on finite sandwich structures with various core sizes gives an understanding of the influence of the boundary layer effects depending on both the architecture and the size of the core. For this validation step, normal compression and simple shear in the transverse plane of the tubes were considered as loading cases.

2. Periodic homogenisation analysis of cellular structures

In order to study the macroscopic mechanical behaviour of heterogeneous solids with specific architectures such as those of the tube stackings considered here, a characterisation procedure is proposed involving both the finite element (FE) modelling of virtual multi-axial loading tests and periodic homogenisation techniques.

2.1. Homogenisation method

This kind of cellular structure can be analysed by following a standard homogenisation method. Therefore, the macroscopic strain $\underset{\sim}{E}$ and the macroscopic stress $\underset{\sim}{\Sigma}$ are introduced, according to the definition of Hill [47] and Suquet [48] for periodic homogenisation. They derive from the microscopic strain $\underset{\sim}{\varepsilon}$ and the microscopic stress $\underset{\sim}{\sigma}$ obeying Eqs. (1) and (2):

$$\underset{\sim}{\Sigma} = \langle \underset{\sim}{\sigma} \rangle = \frac{1}{V_\Omega} \int_{V_\Omega} \underset{\sim}{\sigma}\, dV_\Omega \tag{1}$$

$$\underset{\sim}{E} = \langle \underset{\sim}{\varepsilon} \rangle = \frac{1}{V_\Omega} \int_{V_\Omega} \underset{\sim}{\varepsilon}\, dV_\Omega \tag{2}$$

with V_Ω denoting the volume of Ω, the domain occupied by the considered unit cell. Note that, if Eqs. (1) and (2) are still valid here because V_Ω contains the porous part of the unit cell, their computation is not so obvious for cellular materials from the finite-element method [49,50]. In fact, whereas the stress in the voids is known (it equals to 0), so that the macroscopic stress can still be computed from Eq. (1), the strain in the voids is unknown. In that case, the macroscopic strain can be computed from the displacements on the corners of the unit cell with suitable multi-point constraints on the remaining parts of the boundaries.

The multi-scale character of the studied cellular structures is used to carry out the averaging procedure; one periodic unit cell is isolated for each stacking pattern as illustrated in solid lines in Fig. 1. Since focus is only on both elastic and plastic mechanical properties here, the periodic unit cells are valid representative volume elements (RVEs) of the considered regular tube stackings [26]. FE simulations were conducted on the RVE of each stacking pattern to characterise their homogenised mechanical behaviour under multi-axial loads. The RVEs were meshed with triangular quadratic elements and the FE code *Z-set* (http://www.zset-software.com/) was used for the simulations. The different modelling assumptions formulated in the present work are listed hereafter.

Owing to the extruded character of the tube stackings only two-dimensional cross sections of the RVEs have been considered