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A modified Fourier–Ritz approach for free vibration analysis of laminated functionally graded shallow shells with general boundary conditions

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ABSTRACT

This paper presents a modified Fourier–Ritz approach for free vibration analysis of laminated functionally graded shallow shells with general boundary conditions in the framework of first-order shear deformation shallow shell theory. The displacement and rotation components of the shells are represented by the modified Fourier series consisted of standard Fourier cosine series and several closed-form auxiliary functions introduced to ensure and accelerate the convergence of the series representation. The material properties are assumed to vary continuously through the thickness according to power-law distribution. Four common types of sandwich functionally graded shallow shells are studied. The bi-layered and single-layered functionally graded shallow shells are obtained as special cases of sandwich shells. By setting groups of boundary springs and assigning corresponding stiffness constants to the springs, different boundary conditions including free, clamped, simply supported, elastic boundaries and their combinations are considered. A comprehensive investigation concerning the free vibration of the laminated functionally graded shallow shells with completely free and simply-supported edges is given. The results show that the present method enables rapid convergence, high reliability and accuracy. Numerous new vibration results for shallow shells with different material distributions, lamination schemes and elastic restraints are provided. Some mode shapes of the shallow shells are depicted. Parameter studies illustrate that changes of boundary conditions, material types, thickness schemes and power-law exponents will affect obviously the free vibration characteristic of the shallow shells. In contrast to most existing techniques, the current method can be universally applicable to a variety of boundary conditions including all the classical cases, elastic restraints and their combinations without the need of making any change to the solution procedure.

1. Introduction

As one of common shell structural elements, the shallow shells are widely used in mechanical, aerospace, marine and civil engineering applications and they can have various types of curvature (e.g., circular cylindrical, ellipsoidal, spherical, hyperbolic paraboloidal) and various types of planforms (rectangular, triangular, trapezoidal, circular, and others). In practical applications, these shallow shells are frequently subjected to dynamic loads. Furthermore, the supported types and assembly techniques are always complicated and variable. Consequently, a thorough understanding of vibration characteristics of the shallow shells with general boundary conditions is of particular significant.

In the past decades, a comprehensive investigation focused on vibration of the isotropic shallow shells has been carried out [1–21]. In these research works, various shell theories were proposed and developed which can be classified into three main categories: thin shallow shell theory [1–12] (e.g., classical shell theory or CST), moderately thick shallow shell theory [13–16] (e.g., first-order shear deformation theory or FSDT) and thick shallow shell theory [17–21] (e.g., higher-order shear deformation theory or HSDT and three-dimensional (3-D) elasticity theory). Apart from the aforementioned shallow shell theories, numerous analytical, semi-analytical and numerical methods were developed as well, such as the wave propagation method [9], finite element method (FEM) [14] and Rayleigh–Ritz method [3–7].

With the development of new industries and modern processes, composite materials are extensively used in many fields of modern

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engineering practices. As a new class of composite materials, the concept of functionally graded materials (FGMs) was proposed by Japanese scientists in 1984. FGMs possess smooth and continuous variation of material properties through the thickness. Such materials can relieve the stress concentration found in laminated composites. Consequently, investigations on FGMs are of great interest for many researchers. Recently, a lot of investigators studied the vibration of shallow shells made of FGMs using aforementioned shallow shell theories [22–31]. Alijani and his co-workers [23] analyzed the nonlinear forced vibration of doubly curved thin shallow shell with a rectangular base and simply supported movable edges in which the Donnell's nonlinear shallow shell theory was used. The static response and free vibration of functionally graded cylindrical shell were studied by Zhao et al. [24] using the element-free kp-Ritz method in conjunction with Sander's first-order shear deformation theory. Kiani et al. [25] presented thermoelastic free vibration and dynamic behavior of functionally graded doubly curved shallow shells with rectangular planform and movable simply supported boundaries using the analytical hybrid Laplace–Fourier transformation and first-order shear deformation theory. Yang and Shen [26] investigated the free vibration and dynamic instability of functionally graded cylindrical shallow shells with simply supported and clamped straight edges subjected to combined static and periodic axial forces in thermal environment, where Reddy's higher-order shear deformation theory and a semi-analytical method were employed. Pradyumna and Bandyopadhyay [27] studied free vibration of cylindrical and spherical shallow shells employing the higher-order shear deformation theory and a C^0 finite element formulation. Free vibration of functionally graded spherical as well as cylindrical shallow shell with all edges clamped and simply supported were investigated by Neves et al. [28] using radial basis functions collection according to higher-order shear deformation theory. Matsunaga [29] presented natural frequencies and bucking stresses of simply supported functionally graded shallow shells (i.e., cylindrical, spherical and hyperbolic paraboloidal) by using two-dimensional higher-order theory. There are some researchers using three-dimensional elasticity theory to analyze thick functionally graded shallow, such as Zahedinejad et al. [30] and Farid et al. [31].

As an extension of functionally graded structures, laminated functionally graded structures are also of great interest for many researchers. A lot of literature on free vibration of laminated functionally graded structures can be found [32–48]. However, it is evident that information available about the vibration of laminated FGM shallow shells is far from complete. In particular, most of previous research effects were restricted to vibration of shallow shells with classical boundary conditions. It is well recognized that there exist some deviations from those classical boundary conditions in engineering applications especially in aircraft, space vehicles and submarine. Consequently, the establishment of a reliable and efficient approach for vibration of laminated FGM shallow shells with general boundary condition including classical and elastic restraints is needed. The aim of this paper is to present a modified Fourier–Ritz approach for free vibration analysis of laminated functionally graded shallow shells with general boundary conditions in the framework of first-order shear deformation shallow shell theory. The displacement and rotation components of shells are represented by the modified Fourier series consisted of standard Fourier cosine series and several auxiliary closed-form functions introduced to ensure and accelerate the convergence of the series representation. The material properties are assumed to vary continuously through the layers thickness. Four common types of sandwich functionally graded shallow shells are studied. The bi-layers and single-layer functionally graded shallow shells are obtained as special cases of sandwich shells. By setting groups of boundary springs and assigning corresponding stiffness constants to the springs, different boundary conditions including free, clamped, simply supported, elastic boundaries and their combinations are considered. A comprehensive investigation focused on free vibration of laminated functionally graded shallow shells with completely free and simply-supported edges is given. The results show that the present method enables rapid convergence, high reliability and accuracy. Numerous new vibration results for shallow shells with different material distributions, lamination schemes and elastic restraints are presented. It is shown that vibration frequencies of laminated FGM shallow shells are strongly influenced by the boundary conditions. The natural frequency parameters increase when higher restraining boundary is used. Parameter studies illustrate that changes of material types, thickness schemes and power-law exponents will affect obviously the free vibration characteristic of the shallow shells. It is obvious that the increasing of the power-law exponents leads to the decrease of the natural frequency parameters of laminated FGM shallow. It is also shown that the thickness schemes have more complex influence on the natural frequency of shallow shells.

2. Theoretical formulations

2.1. Description of model

Consider a laminated functionally graded doubly curved shallow shell of rectangular planform with uniform thickness h, as shown in Fig. 1. The shallow shell is characterized by its middle surface where the coordinate system composed of x, y and z is located. The middle surface is described as

$$z=-\frac{1}{2R_x}\left(x-\frac{a}{2}\right)^2-\frac{1}{2R_y}\left(y-\frac{b}{2}\right)^2 \tag{1}$$

where a and b are the length and width of shell planform. R_x and R_y respectively denote the radii of curvature in the x and y directions used to determine the configuration of shell. $\overline{u}$, $\overline{v}$ and $\overline{w}$ are the

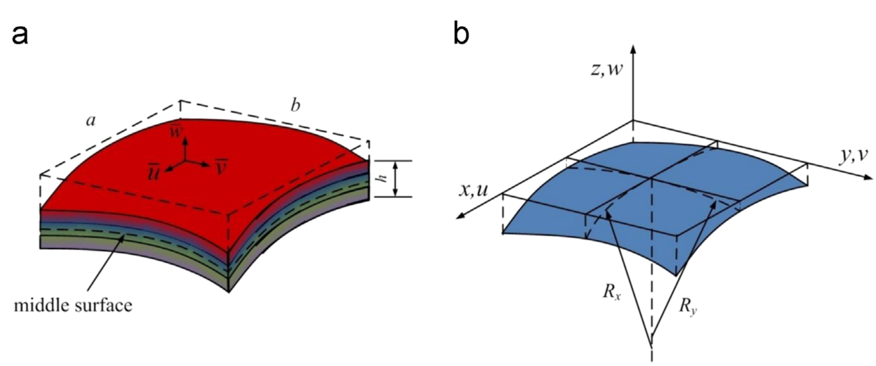

Fig. 1. Geometry of an laminated FGM shallow shell and coordinate.