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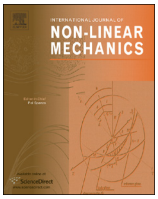

Non-linear stability of under-deck cable-stayed bridge decks

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ABSTRACT

The stability of comparatively more slender decks of under-deck cable-stayed bridges is studied, by considering both the critical loads and the post-buckling behaviour. A potential energy approach is applied to a simplified discrete link and spring model that allows for an exact non-linear formulation of the equilibrium equations. The physical response is found to be dependent on the ratio of the axial stiffness of the cable-staying system to the flexural stiffness of the deck. The influence of several parameters is analysed and unstable mode interaction is observed to occur under certain geometric conditions. The presented analytical model is compared with a non-linear finite element model that shows good correlation. Finally, some design criteria and recommendations are suggested, which are relevant for designers of this innovative typology of cable-stayed bridges.

1. Introduction

Under-deck cable-stayed bridges (UDCSBs) are an innovative typology of cable-stayed bridges [1], in which the stay cables are located underneath the deck [2,3]. The stay cables, which are initially prestressed, are self-anchored to the deck and follow a polygonal layout (Fig. 1). The deviation forces generated in the edges of this layout are introduced into the deck by means of struts, consequently providing additional elastic supports to the deck. Hence, depending on the number of struts employed and the initial prestress force, the bending moments acting on the deck can be reduced substantially when compared to a bridge with no cable-supporting system [4].

UDCSBs have been designed and built since the late 1970s, an example of which is shown in Fig. 2. Research focused on these bridges has demonstrated their advantages for medium spans when compared with conventional bridges without cable-staying systems [4]. These advantages include: (1) higher structural efficiency by reducing the flexural response of the deck and enhancing the axial response; (2) significantly higher deck slendernesses leading to a reduction in the structural self-weight, allowing for more sustainable construction; (3) multiple construction solutions; and (4) arguably, more attractive aesthetic characteristics.

However, UDCSBs may present stability problems during the erection stages due to the compression force introduced by the stay cables into the deck at the support sections. Moreover, the vertical forces acting on the highly slender deck, such as the self-weight and deviation forces, may make the deck prone to buckling. As a consequence, the stability of the bridge when the stay cables are being prestressed needs to be studied, primarily to ensure safety during the construction stages, such the higher deck slenderness that can be achieved during the service life of the bridge is secured. In fact, stability issues and highly non-linear behaviour during the construction of several structures with under-deck cable-staying systems have been reported [5–8].

The stability of compression elements has been studied in conventional cable-stayed bridges in considerable depth [9,10]. In UDCSBs, only the critical loads have been obtained for a particular configuration: the double-level cable-staying system [11,12]. However, as far as the authors are aware, the influence of different parameters on the response and the post-buckling behaviour has not been studied. Defining the post-buckling response becomes crucially important when studying the safety of the bridge: a stable response would allow for setting the design load higher than the critical buckling load. However, an unstable response, which is usually a signature for high sensitivity to initial imperfections, implies that the design load has to be set lower than the critical buckling load [13,14]. Nevertheless, critical loads would not normally be reached during the construction of UDCSBs, and the additional load allowance of stable post-buckling paths would lead to greater safety factors under unexpected loading scenarios.

An analytical approach is presented in the current work that allows for an exact formulation of simplified bridge behaviour, by

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employing a methodology based on energy principles [14,15]. A model comprising discrete rigid-links and springs is employed. These rigid-link models have successfully mimicked the behaviour of prestressed stayed columns, in which mode interaction phenomena can be observed under certain circumstances [13,16,17]. The principal advantage of these rigid-link models is that the relatively simple, but non-linear, formulation allows the determination of the influence of various parameters on the response.

After an initial formulation of the perfect case, a particular solution is presented with the aim of demonstrating the practical application of the model. The presented model allows for multiple initial and boundary conditions that can replicate different construction methods. The results are then compared with the results obtained with a finite element model formulated in the commercial code Abaqus [18]. Finally, the discussion of results, some design criteria and general conclusions are presented.

2. Analytical model development

A single-span UDCSB with two struts and a stay cable eccentricity of 10% of the total span is studied due to its structural efficiency [2,4], the total span being consequently divided into three subspans. The struts, which are pinned to the deck to avoid the introduction of moments (as recommended by [4]), bisect the angle between the stay cables at the edges of the polygonal layout, see Fig. 3. Hence, the deviation force generated by the prestressed stay cable follows the direction of the struts introducing, in turn, an axial and a lateral force into the deck. Following the trend from the research into the buckling of columns, the term 'lateral' is employed to refer to any load that acts perpendicularly to the axis of the deck, such as the vertical component of the deviation forces. The following assumptions are made in the simplified analytical model:

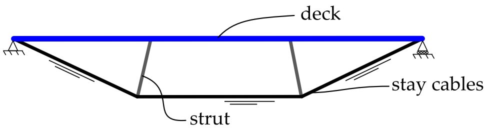

Fig. 1. Elevation of a single-span UDCSB with two struts in conjunction with the main elements: deck, stay cables and struts.

Fig. 2. San Miguelito creek footbridge in Queretaro (Mexico) designed by Carlos Fernandez Casado SL and completed in 2008. Photo courtesy of Arturo Perez Aguilar and Christian Balcazar Benitez (Mexpresa).

1. The axial deformation of the deck and struts is considered to be negligible.
2. A constant flexural rigidity is considered for the entire length of the deck.
3. The cable-staying system is anchored at the centroid of the cross-section of the deck at support sections and therefore no bending moments are introduced into the deck at these sections.
4. All materials and springs are considered to be linearly elastic.

A three degree-of-freedom (3-DOF) link model is presented that allows for the exact formulation of the total potential energy of the system V. Equilibrium equations, which are deduced from V, are solved numerically by means of Auto [19], a powerful a well established numerical continuation package that can compute the bifurcation points as well as the solution branches. The model comprises four rigid links, linear longitudinal springs of stiffness k at pins B and D, and rotational springs of stiffness c at pins B, C and D (Fig. 4). Rotational springs account for the flexural stiffness of the deck, while longitudinal springs represent the cable-staying system. The length of the rigid links is dependent on the parameter α, which is introduced to consider different subspan length distributions.

Even if the stay cables are located purely on one side of the deck, the effect of these can be modelled by means of longitudinal springs such that:

- If a downward perturbation is introduced in the deck, the axial force in the stay cables would increase; consequently, the upward component of the deviation force would increase.
- If an upward perturbation is introduced in the deck, the axial force in the stay cables would decrease; consequently, the upward component of the deviation force would decrease.

There may also be a case of an upward perturbation value where the stay cables slacken, which would diminish the stiffness of the

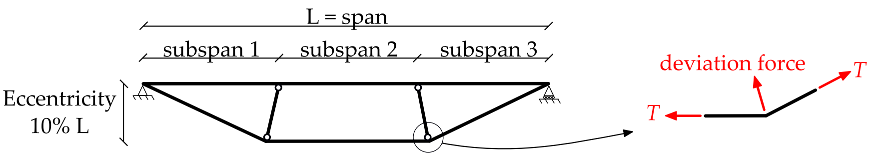

Fig. 3. UDCSB with two struts, a stay cable eccentricity of 10% and the corresponding subspans. The deviation force is a consequence of the prestressing force T in the stay cables.

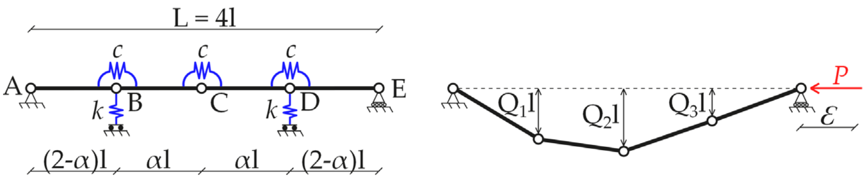

Fig. 4. Rigid-link and springs model with the corresponding rotational and longitudinal springs being of stiffness c and k respectively. Generalized coordinates Q_1, Q_2 and Q_3 define the system kinematics and represent the non-dimensional lateral displacements of nodes B, C and D respectively; $\mathcal{E}$ is the end-shortening of the deck.