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Large-deflection and postbuckling of beam-columns with non-linear semi-rigid connections including shear and axial effects

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A B S T R A C T

The non-linear large deflection-small strain analysis and post-buckling behavior of an out-of-plumb Timoshenko beam-column of symmetrical cross section subjected to end loads (forces and moments) with non-linear bending connections at both ends, and its top end partially restrained against transverse and longitudinal translations are developed in a classical manner. A set of non-linear equations based on the "modified shear equation" that includes the effects of (1) shear deformation and the shear component of the applied axial forces; and (2) shortening of the beam-column due to both axial forces and "bowing" are presented. The proposed method and corresponding equations can be used in the large deflection-small strain analysis of Timoshenko beam-columns with non-linear bending connections, as well as lateral and longitudinal non-linear restraints at the top end. This paper is an extension of previous work presented by the senior author on the large deflection and post-buckling behavior of Timoshenko beam-columns with linear elastic semi-rigid connections and linear elastic lateral bracing. Three comprehensive examples are included that show the effectiveness of the proposed method and corresponding equations. Results obtained in the three examples are verified against analytical solutions available in the technical literature and against results from models using the FEM program ABAQUS.

1. Introduction

The non-linear large deflection-small strain analysis and post-buckling behavior of beam-columns is of great importance in structural engineering, especially for members made of materials with high resilience and low shear stiffness as composite materials. For structures made of this type of materials, the combined influence of lateral deflections along the element chord (i.e., P–δ effects), the relative drift between the ends of the member (i.e., P–Δ effects), axial shortening due to both axial loads and "bowing", softening of the end connections and lateral bracing system, produce a significant non-linear behavior that must be considered in the analysis of these structures.

In the past several years, a significant number of studies have been conducted to understand the large deflection and post-buckling behavior of shear deformable columns. Different applications where these theories are used involve, for instance, the analysis of nano- and micro-beams [1,2], and laced built-up columns [3]. Some of these studies and additional applications are summarized next.

Ansari et al. [1] developed a continuum model for the post-buckling analysis of nanobeams incorporating the effects of surface stress. In their formulation, different boundary conditions were taken into account (e.g., simply-supported, and clamped boundary conditions). Although their formulation is based on a continuum model, their solution involves a discretization in the domain of the beam, which is then solved via a pseudo arch-length method.

Chaterjee and Pohit [2] investigated the static and dynamic large deflection behavior of electrostatically actuated micro-cantilever beams, where geometric and inertial non-linearities were incorporated. The solution of the governing equations was approximately solved using the Galerkin method.

Gantes and Kalochairetis [3] developed a framework for the approximate second order analysis of an imperfect Timoshenko beam-column based on the finite element method. This research focused on the analysis of laced built-up columns, in which effects of shear deformations are significantly important in the stability analysis.

Hjiaj et al. [4] developed a non-linear finite element formulation for the large-displacement analysis of shear-deformable two-layer beams. In this formulation, interlayer slip was allowed and transverse shear deformation of each layer was incorporated. According to their results, deflections in short beams were influenced by shear deformations, obtaining an increase in deflections of 14% for a length-to-depth ratio of six.

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Nomenclature

A and A_s cross-sectional area and effective shear area of beam-column, respectively
E and G elastic and shear modulus of the material respectively
F_{uh} ultimate load capacity of the lateral bracing at the column top end A
F_{uv} ultimate load capacity of the axial restraint at the column top end A
H_z "bowing" bending functions given by Eqs. (5) and (6)
h initial height of the beam-column AB
h_r chord length of the beam-column AB after "bowing" caused by bending and shear actions
I moment of inertia of the beam-column cross-section about its principal axis of bending
k_h and k_v stiffness of the lateral and axial linear restraints at end A, respectively
k_{hi} and k_{vi} initial stiffness of the lateral and axial linear restraints at end A, respectively
M external bending moment applied at A
M_a and M_b end moments of beam-column at A and B respectively
M_{ua} and M_{ub} ultimate moment applied at ends A and B, respectively
n_a and n_b shape parameter of the non-linear bending connections at A and B, respectively
n_h and n_v shape parameter for the lateral and axial non-linear restraints at end A, respectively
P applied vertical load at A (+ compression, − tension)
P_e $\pi^2 EI/h^2$ = Euler load
P_h transverse force developed in the non-linear bracing due to lateral deflection of end A of the beam-column
P_t total axial load along the cord AB (+ compression, − tension)
P_v vertical force in the non-linear restraint due to vertical deflection at end A of the beam-column
Q applied transverse load at A
$u(x)$ lateral deflection of the column center line at x
$\psi(x)$ rotation of cross section of the beam-column at x
κ_{ia} and κ_{ib} initial stiffness of the non-linear bending connections at ends A and B, respectively
ρ_{ia} and ρ_{ib} initial fixity factors of the non-linear bending connections at ends A and B, respectively
θ_0 initial out-of-plumb angle of the cord AB with respect to the vertical axis (Fig. 1a)
θ angle of the cord AB with respect to the vertical axis after loading (Fig. 1a)
ψ_a and ψ_b rotations of cross sections at A and B due to bending, respectively
Δ_h and Δ_v horizontal and vertical deflections of top end A of beam-column AB, respectively

Nguyen and Gan [5] proposed a finite element model for the large deflection analysis of tapered functionally graded beams. The large deflection analysis was performed using a Newton–Raphson technique combined with the arch-length method. Santos and Gao [6] presented a canonical dual mixed finite element method for the post-buckling analysis of elastic beams undergoing large deformations.

Yu et al. [7] presented the numerical and analytical approximate solutions for the electromechanical post-buckling analysis of an actuator fixed at the ends and subjected to a symmetric electrostatic field. The numerical and analytical results showed good agreement for a large range of beam deflections.

Ansari et al. [8] studied the thermal post-buckling behavior of functionally graded Timoshenko microbeams. Stability equations were derived based on the Timoshenko beam theory and the method of virtual work. The effects of imperfections were also discussed, and the behavior of a theoretically perfect beam was compared with the behavior of a beam with initial imperfections.

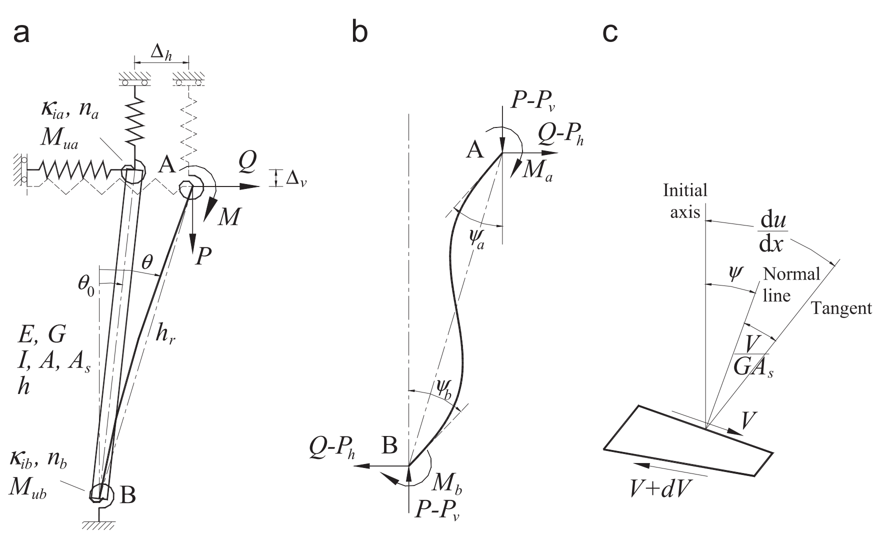

Fig. 1. Beam-column with connections at both ends and with initial out-of-plumb θ_o: (a) structural model; (b) end forces, moments and rotations; and (c) deformed differential element.