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Non-linear dynamic instability of a double-sided nano-bridge considering centrifugal force and rarefied gas flow

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ABSTRACT

Double-sided electromechanical nano-bridges can potentially be used as angular speed sensors and accelerometers in rotary systems such as turbine blades and vacuum pumps. In such applications, the influences of the centrifugal force and rarefied flow should be considered in the analysis. In the present study, the non-linear dynamic pull-in instability of a double-sided nano-bridge is investigated incorporating the effects of angular velocity and rarefied gas damping. The non-linear governing equation of the nanostructure is derived using Euler-beam model and Hamilton's principle including the dispersion forces. The strain gradient elasticity theory is used for modeling the size-dependent behavior of the system. The reduced order method is also implemented to discretize and solve the partial differential equation of motion. The influences of damping, centrifugal force, length scale parameters, van der Waals force and Casimir attraction on the dynamic pull-in voltage are studied. It is found that the dispersion and centrifugal forces decrease the pull-in voltage of a nano-bridge. Dynamic response of the nano-bridge is investigated by plotting time history and phase portrait of the system. The validity of the proposed method is confirmed by comparing the results from the present study with the experimental and numerical results reported in the literature.

1. Introduction

In recent decades, ultra-small beam-type structures have found extensive applications in the fabrication of micro/nano-electromechanical systems (MEMS/NEMS) [1–3] and many researchers have focused on theoretical investigation of the electromechanicl performance of such systems. Among developed NEMS, the double-sided actuating nano-bridges are the most promising elements in constructing speed sensors [4] and new accelerometers in rotary systems such as vacuum pumps and turbine blades. A double-sided actuated nano-bridge shown in Fig. 1 is constructed from a double-clamped movable beam suspended between two fixed electrodes (grounded). Applying voltage between the electrodes leads to deformation of the movable beam towards the fixed electrode. When the electrostatic force exceeds the elastic resistance of the beam, the instability occurs and the movable electrode suddenly adheres to the ground. The stability of micro/nano-bridge has been theoretically investigated by many researchers [5–10]. Ouakad et al. studied the stability and dynamics of doubly-clamped micro-scale structures such as micromachined arches resonators [11–13] and carbon nanotubes (CNTs) [14,15] under DC and AC actuations.

When the distance between electrodes in micro/nanoelectromechanical system becomes of the order of a few microns to nanometer, the van der Waals (vdW) and Casimir forces originated from quantum mechanics, should be taken into consideration [16]. Many researchers investigated the dynamics and stability analysis of micro-scale systems in the absence [17,18] and presence [19,20] of such dispersion forces. The influence of van der Waals (vdW) and Casimir forces on the stability of the electrostatic torsional NEMS has been studied by Guo and Zhao [21]. They demonstrated that in the absence of any actuation, when the gap is sufficiently small, pull-in can still take place because of the action of vdW and Casimir torques. Lin and Zhao [22] presented the analytical expression of the critical pull-in gap with Casimir force using the perturbation theory and showed that the detachment length of the mentioned NEMS increases by increasing thickness of the nano-beam. Sedighi et al. [23] investigated the dynamic instability of functionally graded (FG) nano-bridges considering Casimir attraction and asymptotically obtained the expression for the fundamental frequency of the system using Parameter Expansion

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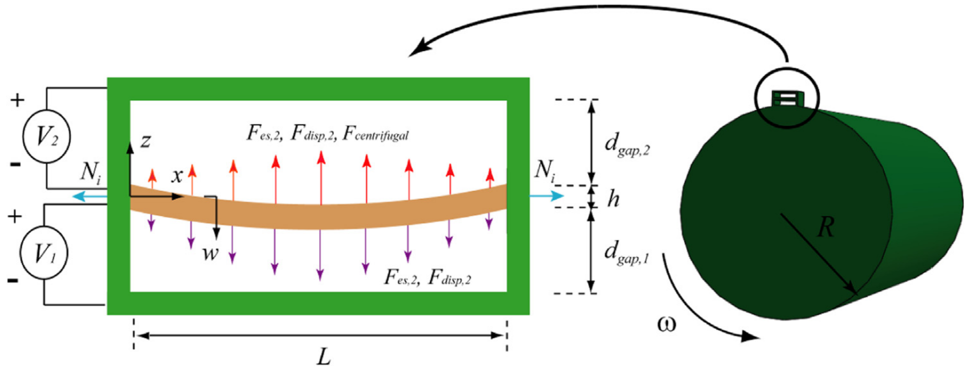

Fig. 1. Schematic representation of double-sided actuated nano-bridge mounting on a rotating shaft under the influence of dispersion and centrifugal forces.

Method (PEM). In another research [24], they presented the modified model for instability analysis of symmetric FGM NEMS and developed a new formulation for Casimir and electrostatic forces to incorporate the impact of finite conductivity of such composite materials.

The nano-bridge accelerometers can be employed as angular speed sensors in fault detection of rotating element bearings with the expected goal to reduce downtime of machines, shaft crack detection of power plant rotating equipments, measurement of high-speed spindle errors in CNC and condition monitoring of electric motors. For these applications, the presence of centrifugal force and rarefied flow should be considered in the analysis. The centrifugal force plays an important role in the mechanical performance of rotary systems [25]. When the nano-bridge sensor is mounted on the rotating shaft, the centrifugal forces change the deflection of the movable electrode. This can significantly affect the pull-in behavior of the nano-bridge and makes the system sensitive to the angular speed of such rotary machines.

In many rotary systems such as gas centrifuge and vacuum rotary pumps, the presence of rarefied gas flow damps the kinetic energy of the movable elements. It is well-known that the prediction and modeling of gas forces on NEMS accelerometers is crucial for reliable design of such structures surrounded by the low pressure gas flows. Chaterjee and Pohit [26] presented the gas damping characteristics of electrostatically actuated microcantilevers in different ambient pressure. McCarthy et al. [27] modeled a cantilever switch considering the effect of gas pressure distribution. Krylov and Maimon [28] assumed the flow in the gap to be incompressible and studied the damped transient characteristics of a cantilever coupled to a plate at the free end. Guo and Alexeenko [29] proposed a new compact model of squeeze-film damping based on the numerical solution of the Boltzmann kinetic equation. Squeeze-film effects of perforated plates for small amplitude vibration through modified Reynolds equation (MRE) have been analyzed by Feng et al. [30]. They found that including the air compressibility is necessary for high operating frequency and small ratio of the plate width to the attenuation length.

The instability analysis of nano-structures on the basis of strain gradient theory can be found in the literature [31–34]. In the present study, the non-linear dynamic pull-in instability of a double-sided nano-bridge as an angular speed sensor is investigated. The effects of centrifugal force and rarefied flow are included in the governing equation of motion. Two fundamental contributions of this work are the presence of rarefied gas flow modeled via the non-linear damping relation and including the centrifugal force that exists when the nano-bridge is mounted on the circumference of a rotating machine, respectively. The strain gradient elasticity theory is also used to investigate the dynamic pull-in instability of an electrically double-sided actuated nano-bridge considering rarefied gas effects and centrifugal forces. Since the mechanical behavior of mirco- and nano-structures is size dependent, the strain gradient theory has been applied to analyze the mechanical behavior of the nano-bridge.

The paper is organized as follows. In Section 2, the governing non-linear equation of the nano-bridge is derived via the Euler beam model. In Section 3, the reduced order method is applied to separate the spatial and the temporal dependence of constitutive partial differential equation (PDE). The solution to the system of ordinary differential equations is numerically obtained using Runge–Kutta method and discussed in Section 4. The influence of length scale parameters is investigated on the dynamic pull-in voltage. Furthermore, the impact of voltage, dispersion and centrifugal forces and damping parameters are demonstrated on the dynamic behavior of vibrating nano-bridge via phase portrait of the system.

2. Theoretical model

2.1. Fundamentals of strain gradient theory

Experiment has indicated that the mechanical behavior of mirco- and nano-structures are highly size dependent [35,36]. For instance, the torsional hardening of copper wire increases by a factor of 3 as the wire diameter decreases from 170 to 12 μm [37]. The bending rigidity of silica and polythene beams can increase significantly when the breadth of the beam reduces to several ten nanometers. Further discussion can be found in Refs. [38–40]. It is worth noting that the classical continuum theories break down to predict and interpret the size effect phenomena [41]. In this regard, size-dependent continuum theories have been developed by introducing additional material length scale parameters as well as the Lame constants [42–45]. One of the most successful size-dependent theories is the strain gradient elasticity theory proposed by Lam et al. [46]. This non-classic continuum theory is more general than the modified couple stress theory [47] and introduces three material length scale parameters to characterize the dilatation gradient tensor, the deviatoric stretch gradient tensor and the symmetric rotation gradient tensor. The material parameters may be determined via experiments or molecular dynamic simulation [38,39]. The strain gradient theory has been applied to analyze the mechanical behavior of ultra-small beams and other structures by many researchers [48–52]. Recently, the strain gradient theory has also been applied for analyzing NEMS [53–58].

In the strain gradient theory, the strain energy density for the linear elastic and isotropic materials with small deformation is written as [47]

$$\overline{U}=\frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij}+p_i\gamma_i+\tau^{(1)}_{ijk}\eta^{(1)}_{ijk}+m^s_{ij}\chi^s_{ij}\right), \tag{1}$$