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Comparison of hyperelastic micromorphic, micropolar and microstrain continua

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ABSTRACT

Micromorphic continua are equipped with additional degrees of freedom in comparison to the classical continuum, representing microdeformations of the material points of a body. Secondary they are provided with a higher order gradient. Therefore, they are able to account for material size-effects and to regularize the boundary value problem, when localization phenomena arise. Arbitrary microdeformations are allowed for in the micromorphic continuum, while the special cases micropolar continuum and microstrain continuum merely allow for microrotation and microstrain, respectively. Amongst these cases, the micropolar case has been covered most extensively in the literature. One goal of this paper is to make the transition from a full micromorphic continuum to a micropolar or microstrain continuum, by varying the constitutive equations. To this end two different possibilities are presented for hyperelasticity with large deformations. This leads to four different material models, which are compared and illustrated by numerical examples. Another goal is to present a constitutive model encompassing the micromorphic, micropolar and microstrain continua as special cases and enabling arbitrary mixtures of micropolar and microstrain parts, allowing the representation of versatile material behaviour.

1. Introduction

Many materials show size-dependent behaviour, e.g. for metals and ceramics the indentation hardness increases with decreasing indenter size for micron-size indents, see [27,21,24,1]. Additionally, there are localization phenomena, such as shear bands, which occur under softening. The classical continuum theory is not able to account for these phenomena, which is why different extensions of classical continuum theory have been developed.

Supplementary quantities are introduced, that lead to non-local behaviour, meaning that the stresses of a material point are dependent on a finite neighbourhood. To govern the non-locality an internal length scale is introduced. The non-local behaviour leads to a regularization of the boundary value problem, when localization phenomena arise, and the internal length scale can be used to represent size dependence.

When higher order gradients of strain or internal variables are used to account for non-local behaviour, the theories are called gradient theories [23,18]. The so-called micromorphic theories, which incorporate micropolar and microstrain theories as special cases, introduce additional degrees of freedom for that purpose. To every material point an additional micro-continuum representing a deformation of this point is attached. Micromorphic models intend to capture the microstructure of a material, by introducing additional degrees of freedom, which are labelled as micro degrees of freedom, but they are still entirely macroscale models. Micromorphic materials were introduced by Eringen in [6]. The subsequent publications, e.g. [7], focussed on micropolar continua, which are a special case of micromorphic continua. Micropolar continua have a rigid micro-continuum, which can only rotate, whereas a micromorphic continuum has fully deformable micro-continua. The micropolar continuum is covered extensively in the literature. Concerning plasticity and parameter identification, see e.g. [26,4,5,22]. Another special case is the microstrain continuum. It is a micromorphic continuum without rigid body rotation of the micro-continua and was introduced in [11]. Plasticity for micromorphic continua has been the subject of various publications in recent years, see e.g. [9,10,12–14,17]. The papers [9,13,14] are also concerned with damage. A comparison of elastic micropolar and micro*stretch* continua for small strains has been conducted in [19].

Micromorphic, micropolar and microstrain continua can be applied to model different materials. For example a hybrid composite of ceramics and polymers consists of deformable

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polymer particles, which are embedded in a ceramic foam. This microstructure can be represented by a micromorphic continuum. Sand on the other hand consists of nearly rigid particles, which are able to rotate against each other. This behaviour can be captured by a micropolar continuum. The microstrain continuum could be useful to model materials with a deformable microstructure with few rigid body rotation, e.g. metal foam.

The material motivating this paper is a cold box sand, as described in [3], which is used for cold box casting. It consists of sand and a binder, which creates a matrix for the sand particles. For the composites of this material different material models would be suitable. For sand, as mentioned before, micropolar models are often considered, because of the nearly rigid particles. In contrast to that, the binder is able to experience strain, and rotation is not an important factor. A micromorphic model should be well suited for a homogenization of these two phases. For this purpose, it seems to be advantageous to have independent influence on the micropolar and microstrain parts of the model.

In this work new material models for the micropolar and microstrain continuum are presented, based on a hyperelastic micromorphic model introduced in [15]. Furthermore, an additive micromorphic continuum model is proposed, which is able to represent the micromorphic, micropolar and microstrain case and combinations of the three by varying the material parameters. In this way it is possible to investigate the influence of the micropolar and microstrain parts with one model in a consistent way. The aforementioned micropolar, microstrain and micromorphic models are used to validate the results of the additive micromorphic model. For this purpose and to illustrate the differences between the material models two numerical examples are presented.

In this paper, Section 2 presents the framework for micromorphic elasticity, with kinematic relations in Section 2.1, followed by balance laws and weak forms in Section 2.2. Hyperelastic constitutive models are introduced in Section 2.3 and the discretization for a finite element implementation is shown in Section 2.4. Section 3 presents numerical examples to compare and illustrate the material models. Finally Section 4 gives a conclusion and an outlook.

2. Micromorphic continuum model

In a micromorphic continuum, the classical macro-continuum is enhanced by adding micro-continua to each material point. These micro-continua may experience affine, kinematically independent deformation, consisting of stretch and rotation, see e.g. [16]. When the deformation is limited to rotation, the continuum is called micropolar, also known as Cosserat continuum. A full micromorphic continuum, as presented in [15] and utilized in this paper, has $(n^{\dim})^2$ additional degrees of freedom, which are called micro degrees of freedom. The components of the so-called micro-deformation map, which represents the deformation of a point in the micro-continuum (as does the deformation gradient for the macro-continuum), are introduced as micro degrees of freedom. This is done instead of introducing micro-displacements, because otherwise C^1-continuity would be necessary for the elements in a finite element implementation, see [20].

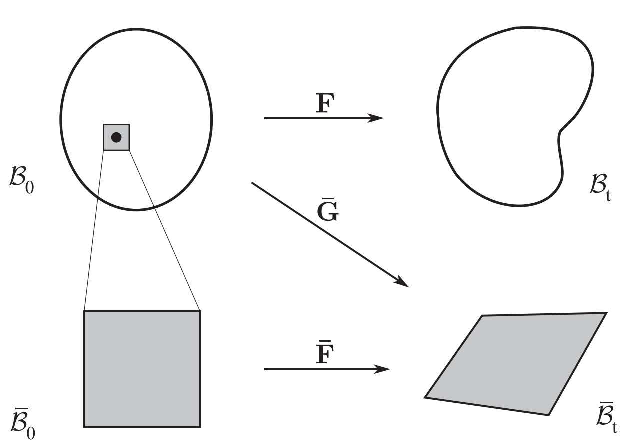

Fig. 1. Kinematic variables.

2.1. Kinematic relations

A material point has coordinates $\mathbf{x}$ in the spatial configuration and coordinates $\mathbf{X}$ in the material configuration. The motion $\boldsymbol{\varphi}$ maps the latter to the former

$$1.\quad \mathbf{x} = \boldsymbol{\varphi}(\mathbf{X}), \qquad 2.\quad \mathbf{F}(\mathbf{X}) = \nabla\boldsymbol{\varphi}(\mathbf{X}). \tag{1}$$

$\mathbf{F}$ is the deformation gradient for the macroscale, with the material gradient represented by ∇.

Additionally, there are micro-continua attached to each macro-material point. To simplify the theory these can only experience homogeneous deformations. Consequently, the deformation gradient of the micro-continua is not dependent on placements $\overline{\mathbf{x}}$ and $\overline{\mathbf{X}}$ of the micro-material points in the spatial and material configuration, respectively. It only depends on the placement of the macro-material points. This is why we skip to introduce a micro-motion and directly introduce a micro-deformation gradient $\overline{\mathbf{F}}(\mathbf{X})$, which is able to represent the deformation of the micro-continuum assigned to $\mathbf{X}$. We also introduce the macro-material gradient of the micro-deformation gradient as

$$\overline{\mathbf{G}}(\mathbf{X}) = \nabla\overline{\mathbf{F}}(\mathbf{X}). \tag{2}$$

The material gradient over the macro-domain is represented by $\nabla(\cdot)$. The kinematic relations are illustrated in Fig. 1 with a body in the material configuration $\mathcal{B}_0$ and the spatial configuration $\mathcal{B}_t$. Additionally, there is a micro-continuum $\overline{\mathcal{B}}$ in both configurations.

2.2. Balance laws and weak form

The local balance of linear momentum in a material form, see e.g. [2, p. 144], for a quasistatic problem is

$$\operatorname{Div}\mathbf{P} + \mathbf{b} = \mathbf{0}, \tag{3}$$

which describes the macroscale. $\operatorname{Div}(\cdot)$ is the material divergence. For a micromorphic continuum we obtain an additional balance of moment of momentum for the microscale, see [8, p. 45]:

$$\operatorname{Div}\overline{\mathbf{Q}} - \overline{\mathbf{P}} = \mathbf{0}. \tag{4}$$

The stresses are of Piola type and denoted as macro-stress $\mathbf{P}$, micro-stress $\overline{\mathbf{P}}$ and micro double stress $\overline{\mathbf{Q}}$, following [15], where the latter is a tensor of third order. $\mathbf{b}$ is the body force. The following boundary conditions, acting in $\mathcal{B}_0$, apply

$$\begin{aligned} &1.\quad \boldsymbol{\varphi} = \boldsymbol{\varphi}^{\mathrm{pre}} \text{ on } \partial\mathcal{B}_0^{\varphi}, \qquad 2.\quad \overline{\mathbf{F}} = \overline{\mathbf{F}}^{\mathrm{pre}} \text{ on } \partial\mathcal{B}_0^{\overline{\mathrm{F}}} \\ &3.\quad \mathbf{P}\cdot\mathbf{N} = \mathbf{t} \text{ on } \partial\mathcal{B}_0^{\mathrm{P}}, \qquad 4.\quad \overline{\mathbf{Q}}\cdot\mathbf{N} = \mathbf{0} \text{ on } \partial\mathcal{B}_0^{\overline{\mathrm{Q}}}. \end{aligned} \tag{5}$$

where $\mathbf{N}$ is the outward normal vector on the material surface and $\mathbf{t}$ is the macro-traction on the Neumann boundary $\partial\mathcal{B}_0^{\mathrm{P}}$. A possible micro-traction on the boundary $\mathcal{B}_0^{\overline{\mathrm{Q}}}$ is neglected, because to the authors' knowledge it cannot be applied in real experiments. A product with one dot denotes a simple contraction. So called strong solutions for every material point with the equations above are generally not available. This is why variational methods are used to gain weak forms. By multiplying Eqs. (3) and (4) with test functions $-\delta\boldsymbol{\varphi}$ and $\delta\overline{\mathbf{F}}$, respectively, and integrating over $\mathcal{B}_0$, we