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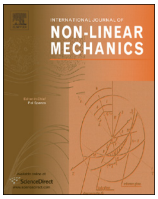

On the dynamics of a micro-gripper subjected to electrostatic and piezoelectric excitations

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ABSTRACT

The aim of this study is to investigate the mechanical behavior of an electrostatically actuated functionally graded piezoelectric micro-gripper. The presented micro-gripper includes two clamped–clamped microbeams symmetrically located against each other in vertical direction. The microbeams are composed of silicon and Piezoelectric Material based on Lead Zirconate Titanate. The mechanical properties, including elasticity modulus, density, and piezoelectricity coefficient are varied exponentially along the height of the microbeam based on power law distribution. The microbeams undergo simultaneous electrostatic and piezoelectric excitations. Two main sources of nonlinearities including geometrical nonlinearity and electrostatic force dominate the dynamics of the gripper. The temporal response is determined by numerically integrating the motion equations and verified by means of method of multiple scales. The contact with the particle is modeled as linear spring which increases the overall stiffness of the grippers. The stability analysis is performed and the pull-in voltage of the device is determined.

1. Introduction

In the recent years, with the increasing effort to minimize the systems and products in industries, the need for micro and nanotechnology has become important issues. The problem of handling the microoptical and microelectrical elements in nano or micrometer range can be solved by using special micro-grippers. A MEMS-based (Micro Electro Mechanical System-based) micro-gripper provides advantages in terms of compact size and low cost and hence plays an important role in microassembly and micromanipulation field for manipulating micromechanical elements, bio-logical cells, etc. Depending on the type of actuation mechanism, micro-grippers can be classified into six types [1].Various prototype of micro-grippers of different actuation methods are developed by numerous researchers, including electro-thermal actuators [2–8], electrostatic actuators [9–14], piezoelectric actuators [15,16], electromagnetic actuators [17], shape memory alloy actuators [18–20], and micro-pneumatic actuators [21]. To sense and measure the tip deflection of the micro-grippers, divers sensors have been presented by authors in the literature including optical sensors [22–24], piezoresistive force sensors [25–30], and capacitive force sensors [31,32]. Electrostatic actuation has been widely applied in MEMS (Micro Electro Mechanical System) devices [33–35] as well as micro-grippers [9–14]. In electrostatic actuators the distance between the microbeam and stationary plate is limited so that applying surplus voltage results in the collapse of the microbeam to the substrate. This phenomenon is known as pull-in and the pertaining voltage is called pull-in voltage. The stability of the micro-gripper is investigated and the pull-in voltage corresponding to various micro-grippers actuated by electrostatic actuation is reported by [36–38]. Many researchers have focused on linear/nonlinear dynamics of the MEMS devices [39–42], similarly MEMS grippers [2–30]. Jahangiri et al. [43] investigated the mechanical behavior of FGP (Functionally Graded Piezoelectric) micro-gripper under DC voltage and temperature variation. It was assumed that the two microbeams of the micro-gripper are mounted symmetrically against each other and the volume of the ceramic varies exponentially with microbeam thickness. They investigated the response and stability of the FGP microbeam against DC voltage and temperature variation.

In this paper, the dynamic and static behavior of the functionally graded piezoelectric (FGP) micro-gripper is studied. Micro-gripper consists of two fully clamped microbeams symmetrically oriented against each other. The mechanical properties of the microbeams including elasticity modulus, density, and piezoelectricity coefficient are graded along the thickness direction according to the power law distribution. The governing differential equation of the motion is derived using Hamiltonian principle and discretized to a single degree of freedom Duffing type ODE (Ordinary Differential Equation) based on Galerkin method. The temporal response and pull-in instability of the device corresponding to various electrostatic voltages and portions of PZT (Piezoelectric Material based on Lead Zirconate Titanate)

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are determined; the temporal response of the device about the static equilibrium position is determined by numerical integration and verified by method of multiple scales. Based on the literature, the contact of the gripper and the particle is modeled as a linear spring.

2. Modeling

As illustrated in Fig. 1, the proposed model is a functionally graded piezoelectric (FGP) micro-gripper including two clamped-clamped micro-beams symmetrically oriented against each other with length ℓ, thickness h, and width a. The mechanical properties are graded along the thickness of the micro-beams obeying power law distribution. The coordinate systems are attached to the lower end mid plane of the microbeams. x is along the length of the beam and z indicates the lateral in-plane direction. The deflection of the microbeams along the z axis is denoted by $w(x,t)$. For a distinct z, the mechanical characteristics of the microbeam are assumed to be a linear combination of the portion of silicon and PZT.

As depicted in Fig. 1, applying V_P to the microbeam, results in the generation of axial stress along the microbeam due to the fixed boundary conditions. The electrostatic voltage is connected to the substrate and lower planes of the micro-beams. The mechanical properties of the micro-beam (elasticity modulus $E(z)$, density $\rho(z)$ and piezoelectric stress constant $e_{31}(z)$) vary with respect to the power law distribution as follows [44]:

$$E(z)=E_0 e^{\frac{2}{h}Ln\left(\frac{E_u}{E_0}\right)|z|}$$

$$\rho(z)=\rho_0 e^{\frac{2}{h}Ln\left(\frac{\rho_u}{\rho_0}\right)|z|}$$

$$e_{31}(z)=e_{31_P}\left(e^{\frac{2}{h}|z|Ln\left(1-P_{P_0}+P_{P_u}\right)}-1+P_{P_0}\right) \tag{1}$$

In Eq. (1), the subscripts "0", and "u" refer to the mid plane and upper plane respectively. P_{P_0} is the portion of piezoelectric material in the mid plane of the microbeam with respect to the silicon satisfying $P_{P_0}+P_{Si_0}=1$. The strain energy due to the bending, axial piezoelectric force and mid-plane stretching, are denoted by U_b, U_P, U_a respectively [45]

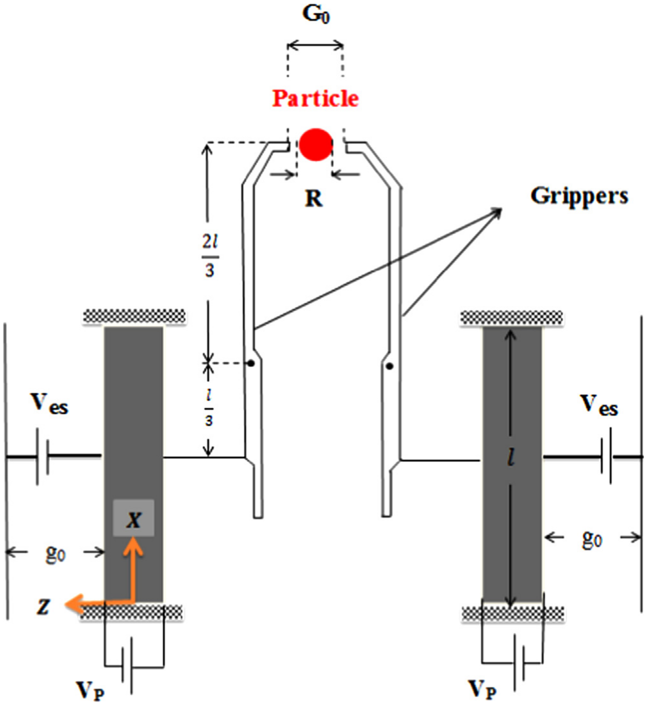

Fig. 1. Schematic view of the FGP micro-gripper exposed to electrostatic and piezoelectric excitation.

U_b is expressed as

$$U_b=\frac{(EI)_{eq}}{2}\int\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{2}$$

where

$$(EI)_{eq}=\int_{-\frac{h}{2}}^{+\frac{h}{2}}E(z)z^2 adz \tag{3}$$

Due to the immovable edges, the extended length of the beam (ℓ') become less than the initial length (ℓ) and this leads to the introduction of axial stress and accordingly an axial force denoted as [46]

$$F_a=\frac{(EA)_{eq}}{\ell}(\ell'-\ell)\approx\frac{(EA)_{eq}}{2\ell}\int_0^{\ell}(\frac{\partial w}{\partial x})^2 dx \tag{4}$$

where

$$(EA)_{eq}=\int_{-\frac{h}{2}}^{+\frac{h}{2}}E(z)adz \tag{5}$$

U_a is expressed as

$$U_a=\frac{1}{2}F_a(\ell'-\ell) \tag{6}$$

Piezoelectric actuation due to the immovable boundary conditions leads to the introduction of another axial force; based on the constitutive equation of piezoelectricity, [47] and considering the direction of the applied electrical field, the axial force due to the piezoelectric actuation is expressed as follows:

$$F_P=\frac{V_P a}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}}e_{31}(z)da \tag{7}$$

where e_{31} is the corresponding piezoelectric voltage constant (Coulomb/m^2), and defined as Eq. (1).

Considering the strain potential energy ($U_P=F_P(\ell'-\ell)$) due to the axial piezoelectric force, the total potential energy of the system reduces to

$$U=U_b+U_a+U_P \tag{8}$$

The kinetic energy of the micro-beam is represented as

$$T=\frac{(\rho ah)_{eq}}{2}\int_0^{\ell}(\frac{\partial w}{\partial t})^2 dx \tag{9}$$

where

$$(\rho ah)_{eq}=\int_{-\frac{h}{2}}^{+\frac{h}{2}}a\rho(z)dz \tag{10}$$

where $\rho(z)$ is defined as Eq. (1)

The work of the electric force from zero deflection to $w(x)$, is expressed as follows:

$$w_{el}=\int_0^{\ell}(\int_0^{w}\frac{\varepsilon_0 aV_{DC}^2}{2(g_0-w)^2}d\zeta)dx=\frac{\varepsilon_0 aV_{DC}^2}{2}\int_0^{\ell}(\frac{1}{g_0-w}-\frac{1}{g_0})dx \tag{11}$$

where g_0 is the initial is gap between the micro-beam and the substrate and ε_0 is dielectric constant of the gap medium and ζ is a dummy parameter. As depicted in free body diagram of the grippers (Fig. 2), once the grippers contact with the particle the force ($2F_C$) conveys to the microbeam at ($x=1/2$), the work of this