International Journal of Non-Linear Mechanics 77 (2015) 265–273

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Displacement dependent pressure load for finite deflection of doubly-curved thick shells and plates

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A R T I C L E I N F O

Article history:
Received 16 February 2015
Received in revised form
25 June 2015
Accepted 2 September 2015
Available online 15 September 2015

Keywords:
Pressure
Shells
Plates
Displacement dependent pressure
Large deformations

A B S T R A C T

The formulation of the displacement dependent pressure load for shells of generic shape and plates is derived in the present study in curvilinear coordinates without any approximation and any hypothesis on material properties. Derivation is initially carried out for pressure load on the middle shell surface, and then for load on the external and internal surfaces by making use of a third-order shear and thickness deformation shell theory, which is a significant improvement for thick shells. The explicit formulation in curvilinear coordinates allows immediate implementation in numerical codes. An approximate formulation is also derived and compared to (i) the exact formulation and to (ii) the displacement independent pressure load which is still widely used in the literature. Applications to circular cylindrical shells and rectangular plates are presented. Comparison of results show that only the exact formulation for displacement dependent pressure load can be used for large deformations of shells and plates, while the displacement independent pressure load can be used only in case of small displacements of the middle surface (or middle plane).

1. Introduction

Pressure is commonly used to represent load by pressurized gas, liquid and wind just to cite a few cases. By definition it is displacement dependent in the sense that this load changes with the deformation of the structures. In fact, pressure is orthogonal to the surface on which it is applied, so its local direction changes with the deformation of the surface. At the same time the surface is stretched or shrunk, so the resulting area on which the pressure is applied changes.

Displacement dependent pressure complicates the analysis, so it is sometimes replaced with constant direction distributed load in Lagrangian description (strains and stresses evaluated in the original undeformed configuration), especially in analytical studies. This approximation can lead to different type of accuracy, due mainly to the large or small deformation of the structure. In particular, for shells and plates made of soft materials (linear elastic or hyperelastic materials, like rubbers or biomaterials), displacement dependent pressure load must be used since they usually undergo very large deformations.

The literature approaching the problem of displacement dependent loads is not that vast, considering the importance of the problem. Almost all studies deal with finite element formulation and they use tensor notation and introduce at some point approximations. Koiter [1] derived the virtual work done by pressure for closed thin shells by using the principle that this is given by the product of the external pressure and the volume variation of the region of space enclosed by the middle surface of the shell. He kept the quadratic and cubic geometrically nonlinear terms in the calculation of the volume variation. The same approach was followed by Libai and Simmonds [2] that applied the formulation to closed shells and membranes, specifying the formulation for toroidal shells.

A general investigation of the mathematical properties of configuration-dependent loading is described by Sewell [3]. This study is not applied to shells and plates or uses shell theories.

Mang [4] derived the displacement dependent pressure stiffness matrices for shells in approximated way for finite element implementation. Argyris and Symeonidis [5] approached the problem of nonconservative (follower) forces by finite element formulation. In particular, the load correction stiffness matrix that represents the nonsymmetric contribution of the configuration-dependent nonconservative loading to the tangent stiffness matrix of the element was derived. Chang and Sawamiphakdi [6] were able to study load due to follower forces by using a finite element incremental scheme. Schweizerhof and Ramm [7] derived the expression of the virtual work done by displacement dependent

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http://dx.doi.org/10.1016/j.ijnonlinmec.2015.09.007
0020-7462/

pressure for finite element incremental loading algorithm. The solution is obtained by incremental decomposition omitting nonlinear terms in the displacement increment.

Cheung and Zhu [8], while studying the postbuckling analysis of circular cylindrical shells under external pressure, derived the exact expression of the external virtual work done by displacement dependent pressure for finite element formulation. They expressed their formulation also in curvilinear coordinates for circular cylindrical shells, but not for shells with variable curvature. Zhu and Cheung [9] extended their study to postbuckling analysis of circular cylindrical shells under combined load, reporting the same derivation of the external virtual work done by displacement dependent pressure in curvilinear coordinates. The final expression (13) in [9] differs in one term with respect to their previous formulation, probably due to a typing error in Eq. (43) in [8].

Even in recent years most of the literature that is not based on finite element formulation still uses displacement independent pressure load, see e.g. [10]. Even if derivations for displacement dependent pressure load for shells are present in the literature, it seems that an exact formulation for doubly curved shells is not present as well as for thick shells with pressure applied on the internal or external surface.

The formulation of the displacement dependent pressure load for shells of generic shape is derived in the present study in curvilinear coordinates without any approximation and any hypothesis on material properties (i.e. it is valid also for hyperplastic materials). This allows immediate implementation in numerical codes. The derivation, not being based on the variation of the enclosed volume, is valid also for open shells and plates. A formulation for pressure applied on the external or internal shell surface instead of the middle plane is also obtained, which is a significant improvement for thick shells. An approximate expression is also derived and compared to (i) the exact formulation and to (ii) the displacement independent pressure load which is still widely used in the literature. Applications to circular cylindrical shells and rectangular plates are presented. Comparison of results show that only the exact formulation for displacement dependent pressure load can be used for large deformations of shells and plates, while the displacement independent pressure load can be used only in case of small displacements of the middle surface (or middle plane).

2. Virtual work by displacement dependent pressure

Pressure is a specific type of load that depends on the deformation of the shell in Lagrangian description. Here for simplicity the pressure is assumed to be applied to the middle surface of the shell. In the next section, the pressure is considered applied to the external or internal shell surface.

A shell of arbitrary shape and material is considered; the theory is valid for shells made of isotropic, laminated composite or functionally graded materials, as well as hyperelastic materials. The shell principal curvilinear coordinates are (α_1, α_2). The displacements of an arbitrary point of coordinates (α_1, α_2) taken on the middle surface of the shell are denoted by u, v and w, in the α_1, α_2 and z directions, respectively; these displacements do not have to be small. The normal displacement w is taken positive outward from the center of the smallest radius of curvature as shown in Fig. 1. In the figure R_1 and R_2 (functions of the coordinates α_1 and α_2) are the principal radii of curvature in α_1 and α_2 directions, respectively.

Initial geometric imperfections of the shell associated with zero initial tension are denoted by displacement w_0 in z direction, also taken positive outward and measured from the ideal middle surface. Imperfections other than in normal directions are not considered and an initial stress-free state is assumed.

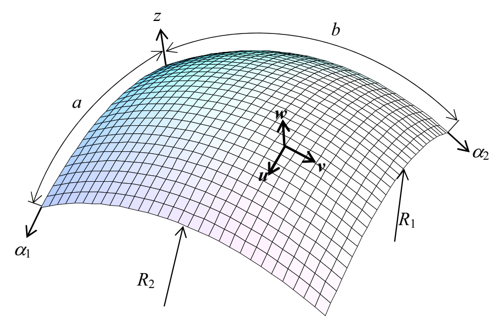

Fig. 1. Displacements of the middle surface of the shell and coordinate system.

The thickness h of the shell is assumed to be small compared to the principal radii of curvature of the shell, but not very small, so that moderately thick shells can be considered with accuracy.

Using the curvilinear coordinates, the middle surface of the shell can be described by the vector equation

$$\mathbf{r} = \mathbf{r}(\alpha_1, \alpha_2) \tag{1}$$

After deformation, the middle surface is given by [11]

$$\mathbf{r}' = \mathbf{r}'(\alpha_1, \alpha_2) = \mathbf{r}(\alpha_1, \alpha_2) + u\mathbf{e}_1 + v\mathbf{e}_2 + w\mathbf{e}_n, \tag{2}$$

where $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_n$ are the three unit vectors of the undeformed shell surface in directions α_1, α_2 and z, respectively. The three unit vectors of the undeformed surface are defined as [11,12]

$$\mathbf{e}_1 = \frac{1}{A_1}\frac{\partial \mathbf{r}}{\partial \alpha_1}, \quad \mathbf{e}_2 = \frac{1}{A_2}\frac{\partial \mathbf{r}}{\partial \alpha_2}, \quad \mathbf{e}_n = \mathbf{e}_1 \wedge \mathbf{e}_2 \tag{3}$$

where A_1 and A_2 are the Lamé parameters. Similarly, after deformation, the unit vectors become

$$\mathbf{e}_1' = \frac{1}{A_1'}\frac{\partial \mathbf{r}'}{\partial \alpha_1}, \tag{4a}$$

$$\mathbf{e}_2' = \frac{1}{A_2'}\frac{\partial \mathbf{r}'}{\partial \alpha_2}, \tag{4b}$$

$$\mathbf{e}_n' = \mathbf{e}_1' \wedge \mathbf{e}_2' \tag{4c}$$

where A_1' and A_2' are the Lamé parameters after shell deformation. The following two relationships follow [11]

$$\frac{1}{A_1}\frac{\partial \mathbf{r}'}{\partial \alpha_1} = (1+\overline{\varepsilon}_1)\mathbf{e}_1 + \overline{\varpi}_1\mathbf{e}_2 - \overline{\Theta}\mathbf{e}_n \tag{5a}$$

$$\frac{1}{A_2}\frac{\partial \mathbf{r}'}{\partial \alpha_2} = \overline{\varpi}_2\mathbf{e}_1 + (1+\overline{\varepsilon}_2)\mathbf{e}_2 - \overline{\Psi}\mathbf{e}_n \tag{5b}$$

where $\overline{\varepsilon}_1$, $\overline{\varepsilon}_2$, $\overline{\varpi}_1$, $\overline{\varpi}_2$, $\overline{\Theta}$, and $\overline{\Psi}$ are defined by [11]

$$\overline{\varepsilon}_1 = \left(\frac{1}{A_1}\frac{\partial u}{\partial \alpha_1} + \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}v + \frac{w}{R_1}\right), \tag{6a}$$

$$\overline{\varepsilon}_2 = \left(\frac{1}{A_2}\frac{\partial v}{\partial \alpha_2} + \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}u + \frac{w}{R_2}\right), \tag{6b}$$

$$\overline{\varpi}_1 = \left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha_1} - \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}u\right), \tag{6c}$$

$$\overline{\varpi}_2 = \left(\frac{1}{A_2}\frac{\partial u}{\partial \alpha_2} - \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}v\right), \tag{6d}$$

$$\overline{\Theta} = \left(-\frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} + \frac{u}{R_1}\right), \tag{6e}$$

$$\overline{\Psi} = \left(-\frac{1}{A_2}\frac{\partial w}{\partial \alpha_2} + \frac{v}{R_2}\right). \tag{6f}$$