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Stabilization of underactuated four-link gymnast robot using torque-coupled method

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ABSTRACT

In this paper, we use an underactuated four-link gymnast robot (UFGR) with a passive first joint to model a gymnast on the bar, and consider the stabilization of the UFGR at the straight-up position. First, we introduce a coupled relationship between control torques for the UFGR. It decouples some state variables of the UFGR from others and changes the nonlinear UFGR system into a cascade-connected system. And then, we use an energy-based method to design a stabilizing controller for the zero dynamics of the cascade-connected system. And the triangle lemma guarantees the control objective of the UFGR to be achieved. The torque-coupled method transforms the stabilization of an underactuated four-link manipulator into that of an underactuated two-link acrobot. This makes the structure of the control system simple. Moreover, our proposed control strategy is easy to extend to the stabilization control of other multi-input nonlinear underactuated systems. Simulation results using the characteristics parameters of a gymnast demonstrate the validity of the proposed method.

1. Introduction

An underactuated mechanical system has more degrees of freedom (DOF) than actuators [1]. Among the advantages of this type of system over the fully-actuated type are that it is lighter, less power-hungry, and more flexible. It has attracted the interest of many researchers. And numerous attempts have been made to investigate the control problem it presents [2–5]. However, the complex dynamics and holonomic and nonholonomic behavior of an underactuated system make the problem very challenging.

An underactuated manipulator is important in many industrial applications, such as space exploration, manufacturing processes, and danger avoidance. To extend the range of applications of this type of robot and to deepen our understanding of it, researchers have invented several pendulum-type robotic models including an acrobot [6], a pendubot [7], and a cart-pendulum robot [8]. An acrobot is a planar underactuated two-link manipulator, where the first link and second link are attached to a passive joint and an actuated joint respectively. The control objective of the acrobot is firstly to swing it up from any position and finally to balance it at the upright position [9]. The swing-up problem for the acrobot has been widely discussed. Many methods have been applied to it, such as partial feedback linearization [10], an energy-based method [11,12], a rewinding approach [13], fuzzy control [14], and sliding mode control [15]. Among them, an energy-based method produces comparatively good results. The method ensures that the acrobot enters the attractive area around the upright position with a stretch-out posture. It is advantageous for the quick stabilization of the acrobot. However, as pointed out in [16], this approach is difficult to be employed directly when the DOF of an underactuated manipulator is greater than two.

An acrobot is a highly simplified model of a gymnast on a bar. The passive first joint and actuated second joint model the hands and hips of a gymnast respectively. Since the structure of human body is complex, some studies have modeled a gymnast using a n-link ($n>2$) manipulator to be able to more realistically and precisely mimic the gymnastic exercise on the bar. For example, Takashima [17] and Xie et al. [18] added a joint to model a gymnast's shoulders and gave a model of three-link gymnast robot. Yeadon et al. [19] used four links to model a gymnast's arms, torso, thighs, legs, respectively. This gave a model of four-link gymnast robot. In the past few years, many researchers have studied the

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control problems for an underactuated three-link manipulator in a vertical plane, i.e., underactuated three-link gymnast robot [20–22]. But there are few analysis results about the motion control of the four-link case [23]. This seems to be caused by two reasons. First, the nonlinearity of the system dynamics becomes more complicated as the DOF increases. Second, the control problems for different underactuated systems have different degrees of difficulty. So, a method that is effective for an underactuated two- or three-link manipulator cannot be directly applied to a four-link one. This makes the control of an underactuated four-link manipulator a big challenge.

This paper studies the motion control of an underactuated four-link gymnast robot (UFGR). The first joint of the UFGR is passive and others are actuated. We develop a torque-coupled control method to solve the stabilization problem for the UFGR at the straight-up position from initial position. First, we use a coupled relationship between control torques to decouple some state variables of the UFGR from others. This changes the UFGR to be a cascade-connected system. This also transforms the stabilization of the UFGR into that of an underactuated two-link acrobot. Second, we use an energy-based method to design the stabilizing controller for the acrobot, which exactly is the model of the zero dynamics of the cascade-connected system. The motion control objective of the UFGR is guaranteed by the triangle lemma.

2. Dynamics of underactuated four-link gymnast robot

Fig. 1 shows the model of an underactuated four-link gymnast robot (UFGR) with a passive first joint. For the i th link ($i=1,\ldots,4$), m_i is the mass, L_i is the length, J_i is the moment of inertia around the center of mass (COM), and L_{ci} is the distance from the COM to the ith joint. The motion equations of the UFGR are

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{G}(\boldsymbol{q})=\boldsymbol{\tau}, \tag{1}$$

where $\boldsymbol{q}=[q_1,\cdots,q_4]^T$ is the vector of generalized coordinates; $M(\boldsymbol{q})=[M_{ij}(\boldsymbol{q})]_{4\times 4}$ is a positive-definite, symmetric inertia matrix; $\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})=[H_i(\boldsymbol{q},\dot{\boldsymbol{q}})]_{4\times 1}$ consists of the Coriolis and centrifugal forces; $\boldsymbol{G}(\boldsymbol{q})=[G_i(\boldsymbol{q})]_{4\times 1}$ is the term for gravity; and $\boldsymbol{\tau}=[0,\tau_2,\tau_3,\tau_4]^T$ is the vector of torques applied to the active joints. The components of $M(\boldsymbol{q})$, $\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})$, and $\boldsymbol{G}(\boldsymbol{q})$ are listed in Table 1, where $g=9.80665$ $\mathrm{m/s^2}$ is the gravitational constant.

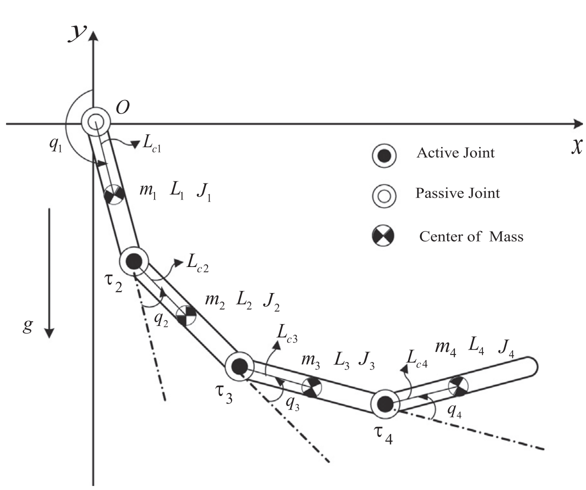

Fig. 1. Underactuated four-link gymnast robot.

Let $x_i=q_i$, $x_{i+4}=\dot{q}_i, i=1,\ldots,4$, $\boldsymbol{x}=[x_1,\cdots,x_4,x_5,\cdots,x_8]^T$. We rewrite (1) in the state space to be

$$\begin{cases} \dot{x}_1=x_5, \\ \dot{x}_2=x_6, \\ \dot{x}_3=x_7, \\ \dot{x}_4=x_8, \\ \dot{x}_5=f_1(\boldsymbol{x})+b_1(\boldsymbol{x})\tau_2+c_1(\boldsymbol{x})\tau_3+d_1(\boldsymbol{x})\tau_4, \\ \dot{x}_6=f_2(\boldsymbol{x})+b_2(\boldsymbol{x})\tau_2+c_2(\boldsymbol{x})\tau_3+d_2(\boldsymbol{x})\tau_4, \\ \dot{x}_7=f_3(\boldsymbol{x})+b_3(\boldsymbol{x})\tau_2+c_3(\boldsymbol{x})\tau_3+d_3(\boldsymbol{x})\tau_4, \\ \dot{x}_8=f_4(\boldsymbol{x})+b_4(\boldsymbol{x})\tau_2+c_4(\boldsymbol{x})\tau_3+d_4(\boldsymbol{x})\tau_4, \end{cases} \tag{2}$$

where

$$\begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) \\ f_4(\boldsymbol{x}) \end{bmatrix} = M^{-1}(\boldsymbol{q}) \begin{bmatrix} -H_1(\boldsymbol{q},\dot{\boldsymbol{q}})-G_1(\boldsymbol{q}) \\ -H_2(\boldsymbol{q},\dot{\boldsymbol{q}})-G_2(\boldsymbol{q}) \\ -H_3(\boldsymbol{q},\dot{\boldsymbol{q}})-G_3(\boldsymbol{q}) \\ -H_4(\boldsymbol{q},\dot{\boldsymbol{q}})-G_4(\boldsymbol{q}) \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} b_1(\boldsymbol{x}) & c_1(\boldsymbol{x}) & d_1(\boldsymbol{x}) \\ b_2(\boldsymbol{x}) & c_2(\boldsymbol{x}) & d_2(\boldsymbol{x}) \\ b_3(\boldsymbol{x}) & c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ b_4(\boldsymbol{x}) & c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix} = M^{-1}(\boldsymbol{q}) \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

The mechanical energy of the UFGR is

$$E(\boldsymbol{x})=\frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}}+\sum_{i=1}^{4}\beta_i\cos(q_1+\cdots+q_i). \tag{5}$$

From (1), we get the derivative of $E(\boldsymbol{x})$ with respect to time variable t

$$\begin{aligned} \frac{dE(\boldsymbol{x})}{dt} &= \dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}}+\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}}-\sum_{i=1}^{4}\beta_i(\dot{q}_1+\cdots+\dot{q}_i)\sin(q_1+\cdots+q_i) \\ &= \dot{\boldsymbol{q}}^T\left[M(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{G}(\boldsymbol{q})\right]+\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &= \dot{\boldsymbol{q}}^T\left[\boldsymbol{\tau}-\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})\right]+\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}}. \end{aligned} \tag{6}$$

It is not difficult to verify that

$$\dot{\boldsymbol{q}}^T\boldsymbol{H}(\boldsymbol{q},\dot{\boldsymbol{q}})-\frac{1}{2}\dot{\boldsymbol{q}}^T\dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}}=0 \tag{7}$$

Combining (6) and (7) gives

$$\frac{dE(\boldsymbol{x})}{dt}=\dot{\boldsymbol{q}}^T\boldsymbol{\tau}=x_6\tau_2+x_7\tau_3+x_8\tau_4. \tag{8}$$

For convenience, we denote

$$\Lambda_1(\boldsymbol{x})=\begin{bmatrix} b_2(\boldsymbol{x}) & c_2(\boldsymbol{x}) & d_2(\boldsymbol{x}) \\ b_3(\boldsymbol{x}) & c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ b_4(\boldsymbol{x}) & c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix}, \tag{9}$$

$$\Lambda_2(\boldsymbol{x})=\begin{bmatrix} b_1(\boldsymbol{x}) & c_1(\boldsymbol{x}) & d_1(\boldsymbol{x}) \\ b_3(\boldsymbol{x}) & c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ b_4(\boldsymbol{x}) & c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix}, \tag{10}$$

$$\Theta_1(\boldsymbol{x})=\begin{bmatrix} f_2(\boldsymbol{x}) & c_2(\boldsymbol{x}) & d_2(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) & c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ f_4(\boldsymbol{x}) & c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix}, \tag{11}$$

$$\Theta_2(\boldsymbol{x})=\begin{bmatrix} f_1(\boldsymbol{x}) & c_1(\boldsymbol{x}) & d_1(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) & c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ f_4(\boldsymbol{x}) & c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix}, \tag{12}$$

$$\Gamma_1(\boldsymbol{x})=\begin{bmatrix} c_3(\boldsymbol{x}) & d_3(\boldsymbol{x}) \\ c_4(\boldsymbol{x}) & d_4(\boldsymbol{x}) \end{bmatrix}, \tag{13}$$