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A hybrid scheme for simulation of planar rigid bodies with impacts and friction using impact mappings

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ABSTRACT

This paper introduces numerical techniques necessary for the implementation of impact maps derived from an energetic impact law for rigid-body impacts with friction at isolated contact points. In particular the work focuses on methodologies for long-term simulation with behaviours such as dynamic transitions and chatter. The methods are based on hybrid event-driven numerical solvers for ordinary differential equations together with system states to deal with the transitions. A slender rod impacting a periodically oscillating surface is used as an example to illustrate implementation and methods. The numerical scheme for the rod system is used to show how symmetry can play an important role in the presence of friction for long-term dynamics. This will show that surface oscillations with low frequencies tend to lead to periodic motions of the rod that are independent of friction. For higher frequencies however the periodic solutions are not that common and irregular motion ensues.

1. Introduction

Collisions or impacts in mechanical systems are very common and in many mechanical engineering applications they can cause undesired wear and noise and thus be very problematic and expensive. Two experimental examples of systems with such issues include an engine cam follower [32] and a magnetic bearing system [22].

In many applications energy is dissipated during impacts through motion in both the normal and tangential direction (friction) and how this happens has wide reaching effects on both the short-term and long-term dynamics. The understanding of the forces and impulses that occur at impact, together with an *impact law*, allows for some prediction of post-impact dynamics of impacting systems. By an impact law we mean a physical law that is based on theory and experimentation and used to describe the physics of a collision between bodies. Many impact law models have been developed, some of which include friction [7,8,11, 21,28,40,18] and some which do not [9,10]. Impact laws can be split into two main classes, those suitable for *compliant* rigid bodies and those suitable for *non-compliant* rigid bodies. The first class of impact laws allows for deformation of the contacting regions of the bodies [5], whereas the second class of impact laws requires perfect rigidity together with some rigidity constraint [8].

The impact law used in this paper will be for non-compliant rigid-body impacts with friction. Typically, the main assumptions for non-compliant impacts are: (i) there is no deformation of the contact regions, (ii) an impact occurs at an isolated contact point, (iii) there is no moment impulse during impact, (iv) the contact duration is infinitesimal, (v) there is no change in generalised coordinates throughout the impact phase and (vi) the finite active forces can be neglected during impact [8,19].

The dynamics of rigid-body systems with impacts and friction is usually found by numerical integration of systems of ordinary differential equations (ODEs) corresponding to the mechanical system under question. There are two main schemes for how this is usually done, namely, *time-stepping* and *event-driven* schemes. How to choose one over the other depends on the class or type of mechanical system that is being analysed, how the impact law is resolved, and the type of numerical analysis one would like to perform. Time-stepping schemes consist of a time discretisation of the dynamics in which each time step is advanced by solving an appropriate *complementarity problem* [1]. In these schemes the moment of each collision or when changes in relative velocity between bodies occurs is not exactly located but instead some level of penetration can occur. This is the price to pay for using rigidly formulated time-stepping methods. These schemes are however very advantageous for the simulation of systems with a large number of degrees of freedom with multiple contacts, for

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example flows in a granular material or masonry structures. Event-driven schemes are also basically time-stepping schemes but the time for which a trajectory reaches a constraint or *discontinuity surface* is located as precisely as possible to avoid penetration. This class of schemes can in turn be divided into two separate categories, the complementarity methods and the *hybrid* methods. As the name suggests, in the complementarity method a complementarity problem is solved as the event is located, whereafter the standard time-stepping scheme continues as discussed above [1].

In the hybrid methods, which are the focus of this paper, the integration is terminated when an event is located and a discrete map is applied to describe how the state changes at the event. When the map is applied the time-stepping scheme is restarted with the post-event states as the initial conditions with a new set of ODEs that reflect the new circumstance. Hybrid methods have some obvious drawbacks but also some very important advantages that we will use in this paper. The main complication of hybrid methods is that, since each event has to be identified and resolved individually, the complexity of all different combinations of events and ODEs grows very quickly with the number of possible events. Another complication is that for each event a mapping has to be found that reflects, in the case of this paper, what the impact law dictates. These issues make hybrid schemes only feasible for systems with relatively few different discontinuity surfaces [2,35,36]. However, on the positive side it is worth raising at least four different points. First, since the events are not included in the time stepping only ODEs with smooth dynamics have to be integrated and it is thus possible to use a suitable high-order integrator with well-known convergence properties so that trajectories can be found with high accuracy. Second, since no events will be lost during simulation hybrid methods are useful for the brute-force bifurcation analysis and in particular when *discontinuity-induced bifurcations* (DIBs) are involved. An example of a DIB, and something that will be seen in this paper, is a *grazing bifurcation*. A grazing bifurcation occurs when, under parameter variation, a trajectory of a periodic orbit makes tangential contact with the discontinuity surface, resulting in a change in the system dynamics [30,15]. Third, hybrid methods make the stability analysis of periodic orbits relatively straight forward since it is possible to calculate *saltation matrices* that "glue" fundamental solution matrices together for trajectories passing through regions between different events. Fourth, despite the common misconception, there are methods born out of hybrid schemes for impacting systems that can deal with the accumulation of impacts, sometimes referred to as *chatter* or *Zeno* behaviour, and also calculate the corresponding saltation matrices [31]. As mentioned above, event-driven schemes are particularly useful for systems with relatively few degrees of freedom, but with multiple spatially and temporally separated contact points. Some examples include turbine blade dampers, friction clutch vibrations, landing gear dynamics [33], passive walkers [2,34] and braille printers [13].

The use of non-smooth system theory to predict and understand the kinematics of colliding rigid bodies in the presence of impact and friction is a useful commodity in engineering in particular and research of such systems in general [6,16,22,25,33,37]. It is well known that non-smooth systems can exhibit complex behaviour that cannot be found in smooth systems. The class of systems with combinations of ODEs and maps, that we use here for mechanical systems with impact and/or friction, are often termed as *piecewise-smooth* (PWS) systems. In recent years the interest of DIBs found in PWS systems has increased dramatically, and as mentioned above the main driver of the analysis of DIBs has been the hybrid system approach, where local behaviour can numerically be pinpointed with high accuracy [3,12,15,30,31]. In particular DIBs in impacting systems without friction have been studied extensively and some classification methodologies have been developed in [14,27,24], but also impacting systems with friction have been studied from a DIB point of view [23,17,28]. As already mentioned, a type of behaviour that is very specific to impacting systems with rigid body impacts is chatter, which is the phenomenon whereby a system goes through an infinite number of impacts in a finite time period. Previous works on chatter have considered both frictionless systems [9,12,31] and systems with impacts and friction [23,28,29]. An interesting example of an engineering-based frictionless system is analysed in [26], where the problem of gear rattle (chatter) in Roots blower vacuum pumps is considered, and where the rattle is induced from the gear teeth losing and regaining contact. Similarly, in cam-follower systems for certain conditions the follower detaches from the cam, resulting in a series of unwanted impacts or chatter [3,4,32].

With this in mind, the emphasis of this paper is two-fold. First, we will show how the impact mappings for impacts with friction derived in [28] can be implemented for reliable simulations of systems with impacts and friction. Second, we want to exploit the fact that we have reliable simulation routines to analyse the long-term qualitative behaviour of an unconstrained mechanical system with impacts and friction. Since previous research has mainly considered long-term dynamics for systems with impacts but without friction [3,14,31] the analysis of the unconstrained object will show that it is feasible to also consider long-term simulations for mechanical system with impact and friction. For this purpose we chose a hybrid event-driven technique as opposed to a time-stepping method in order to resolve DIBs in brute-force bifurcation analysis as well as deal with accumulation of events (chatter). We will use the example of a planar slender rod impacting with an oscillating surface to show how to implement these techniques. This can be seen as a generalisation of a system where a machine element detaches and is free to vibrate in the presence of friction or an item that lies on a vibrating conveyor belt. We will show how rattle is affected by the presence of friction.

This paper is organised as follows. The equations of motion for a collision between two rigid bodies with an isolated contact point are derived in Section 2.2 along with an extension of the energetic impact law derived in [28] to allow for a two-body collision. Section 3 summarises the numerical methods necessary for the implementation of the chosen impact law. In Section 3.1 we will introduce the notion of system states and how this idea is used in the simulation of impacting systems. The model example of a slender rod impacting a periodically oscillating surface is introduced in Section 4 and the basic setup that is used in the numerical simulations is presented in Section 4.1. The paper concludes with a discussion in Section 5 that provides an insight for engineers and other researchers working with impact and friction.

2. Planar rigid-body impacts with friction

In this section we will derive a general framework for a planar rigid-body collision between two unconstrained objects. We will present an extension of the energetic impact mapping derived in [28], which will be used for the model example in Section 4. The extension derived here is more general than the mapping presented in [28] in that the mapping in [28] is for the specific case of a slender rod impacting a stationary non-compliant surface, but where we allow both bodies to be unconstrained.

2.1. Equations of motion

Consider two planar rigid bodies H and H' whose configuration relative to an inertial reference frame can be described in terms of