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Non-local free and forced vibrations of graded nanobeams resting on a non-linear elastic foundation

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ABSTRACT

We consider in this paper the free and forced vibration response of simply-supported functionally graded (FG) nanobeams resting on a non-linear elastic foundation. The two-constituent Functionally Graded Material (FGM) is assumed to follow a power-law distribution through the beam thickness. Eringen's non-local elasticity model with material length scales is used in conjunction with the Euler–Bernoulli beam theory with von Kármán geometric non-linearity that accounts for moderate rotations. Non-linear natural frequencies of non-local FG nanobeams are obtained using He's Variational Iteration Method (VIM) and the direct and discretized Method of Multiple Scales (MMS), while the primary resonance analysis of an externally forced non-local FG nanobeam is performed only using the MMS. The effects of the non-local parameter, power-law index, and the parameters of the non-linear elastic foundation on the non-linear frequency-response are investigated.

1. Introduction

Recently, several investigators have focused their attention on developing structural theories for rods, beams, plates, shells and other structural systems that incorporate the effect of material length scales. Modern devices like microactuators, microswitches, biosensors, nanowires, nanoprobes, ultra thin films, and MEMS and NEMS [1–5] use miniaturized beams as a structural basis. Arash and Wang [6] demonstrated that material properties are size-dependent at the nano-scale thereby reinforcing the necessity to consider small length scale effect when simulating these micro- and nano-structures. Unlike classical continuum mechanics theories, non-local elasticity theories incorporate these size-dependent effects when modeling these structures. The use of such theories has found attractive applications in several areas including fracture mechanics, lattice dispersion of elastic waves, dislocations and wave propagation [7].

A number of size-dependent theories have been put forward in the literature, which include Eringen's non-local elasticity theory [8–11], modified couple stress theory of Mindlin [12], Koiter [13], and Toupin [14], and the strain gradient theory [15–17]. The strain gradient theory and the modified couple stress theory, although different, can be correlated [18]. A limited number of investigators exploited the modified couple stress and the strain gradient theories to model size-effects in studying linear and non-linear static, vibration and buckling response of homogeneous micro- and nano-beams [19–27] and [28,29], respectively. On the other hand, the use of Eringen non-local elasticity differential model has found more popularity among researchers. Reddy [30] reformulated the Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories using Eringen's non-local differential constitutive relations to find analytical solutions for bending and bucking of beams. Thai [31] proposed a non-local shear deformation beam theory to study the bending, buckling and vibration response of a nanobeam. A similar study was conducted by Roque et al. [32] on a Timoshenko nanobeam using a meshless method based on collocation with radial basis. Li et al. [33] used a perturbation method to analytically investigate the natural frequency of vibration, steady-state resonance and stability of a non-local nanobeam with a variable axial load. Eltaher et al. [34] used a non-local finite element model to study the static bending and buckling of functionally graded Euler–Bernoulli nanobeams. The previous studies are all linear problems. Reddy [35] accounted for the von Kármán non-linear strains and reformulated classical and shear deformation beam and plate theories. Similarly, Simsek [36] estimated the non-linear free vibration problem of an Euler–Bernoulli nanobeam, accounting for the von Kármán non-linear strains and solved the associated non-linear ordinary differential equation using He's variational statement [37]. Using the Differential Quadrature Method (DQM), Najar et al. [38]

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studied the pull-in instability of a non-local non-linear response of a DC voltage excited capacitive nanoactuator modeled as an Euler–Bernoulli beam with different boundary conditions. The model accounts for the residual stresses, initial deflection, von Kármán non-linear strains, electrostatic forcing and intermolecular forces, such as Casimir and von der Waals forces.

Functionally graded materials (FGMs) have emerged as a promising alternative to homogeneous coatings [39]. FGMs comprise of at least two-phase inhomogeneous particulate composites and are synthesized in such a way that the volume fractions of the constituents vary continuously along any desired spatial direction, resulting in materials having smooth variation of mechanical properties. Such enhancements in properties endow FGMs with qualities such as resilience to fracture through reduction in propensity for stress concentration. FGMs promise attractive applications in a wide variety of wear coating and thermal shielding problems such as gears, cams, cutting tools, high temperature chambers, furnace liners, turbines, micro-electronics and space structures. With the progress of technology and fast growth of the use of nanostructures, FGMs have found potential applications in micro- and nano-scale in the form of shape memory alloy thin films [40], electrically actuated actuators [41], microswitches [42] and atomic force microscopes (AFMs) [43].

With the recent interest in size-dependent models at the micro- and nano-level, there has been an increased focus on characterizing the free and forced vibration of functionally graded micro- and nanobeams using a number of numerical and analytical techniques. Using the finite element method, Eltaher and co-workers [34,44,45] studied the linear static bending, vibration and buckling behaviors of a functionally graded Euler–Bernoulli and Timoshenko nanobeam. Reddy et al. [46] also used the finite element method to model a non-linear non-local functionally graded microbeam to show the effect of the non-local parameter and power law index on deflections and stresses.

In addition to these numerical methods, a number of analytical and semi-analytical methods have been proposed to solve the free vibration of functionally graded microbeams to develop closed-form approximations of the non-linear frequency. Using Navier's method, Uymaz [47] solved analytically the free and forced vibration problem of a graded nanobeam using various classical and higher-order beam theories. A similar study was carried out by Rahmani and Pedram [48] who exploited Navier's method to study analytically the vibration behavior of graded Timoshenko nanobeams. Along similar lines, Simsek and Yurtcu [49] employed Navier's method to perform a bending and buckling analysis of non-local functionally graded Timoshenko beams. Very recently, He's Variational Method was applied by Simsek et al. [36,50] to estimate the non-linear frequency associated with free vibration of non-local functionally graded Euler–Bernoulli and Timoshenko beams. Finally, using the method of multiple scales and Galerkin's method, Nazemnezhad and Hosseini-Hashemi [51] accounted for the non-linear von Kármán strains and studied analytically the non-linear free vibration of functionally graded Euler–Bernoulli nanobeams.

To the best of the authors' knowledge, it can be concluded from the literature review that very few researchers have focused their attention on studying the non-linear non-local free and forced vibration response of homogeneous and graded nanobeams using analytical methods such as Navier's Method [47–49], He's Variational Method [36,50] and method of multiple scales [51]. To date only Hosseini et al. [52] worked on determining the effect of the power law exponent on the frequency response function for the forced vibration analysis of functionally graded beams. However, the study was not conducted at the nanoscale, so therefore does not consider non-local elasticity theories. The present study is intended to fill this gap in the literature by considering the non-local non-linear free and forced vibration response of nanobeams using He's Variational Iteration Method (VIM) and the Method of Multiple Scales (MMS). The forced vibration analysis includes studies related to primary resonance. This paper includes a complete theoretical development of the considered problem and a parametric study which brings out the effects of the power-law index and non-local parameter on the free and forced vibration response.

This paper is organized as follows. Following this introduction, Eringen's non-local differential model is revisited in Section 2. The governing equations for a classical and a non-local graded Euler–Bernoulli beam accounting for moderate rotations are given in Sections 3 and 4, respectively. The free and forced vibration solutions obtained using VIM and the discretized and direct MMS are given in Sections 5 and 6, respectively. Numerical results are provided in Section 7. Finally, the main contributions and conclusions of this study are summarized in Section 8.

2. Eringen's non-local differential model

According to Eringen [8,9], the state of stress $\boldsymbol{\sigma}$ at a point $\mathbf{x}$ in an elastic continuum not only depends on the strain field $\boldsymbol{\varepsilon}$ at the point, as in the case of classical continuum theories, but also on strains at all other points of the body. Eringen attributed this to the atomic theory of lattice dynamics and experimental observations on phonon dispersion. Thus, the non-local stress tensor $\overline{\boldsymbol{\sigma}}$ at point $\mathbf{x}$ is expressed as

$$\overline{\boldsymbol{\sigma}} = \int_{\Omega} K(|\mathbf{x}' - \mathbf{x}|, \tau)\boldsymbol{\sigma}(\mathbf{x}')\, d\mathbf{x}' \tag{1}$$

where $\boldsymbol{\sigma}(\mathbf{x})$ is the classical, macroscopic second Piola–Kirchhoff stress tensor (see Reddy [53]) at point $\mathbf{x}$ and the kernel function $K(|\mathbf{x}'-\mathbf{x}|, \tau)$ represents the non-local modulus, $|\mathbf{x}'-\mathbf{x}|$ being the distance (in the Euclidean norm) and τ is a material parameter that depends on internal and external characteristic lengths (such as the lattice spacing and wavelength, respectively). The macroscopic stress $\boldsymbol{\sigma}$ at a point $\mathbf{x}$ in a Hookean solid is related to the strain $\boldsymbol{\varepsilon}$ at the point by the generalized Hooke's law

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : \boldsymbol{\varepsilon}(\mathbf{x}) \tag{2}$$

where $\mathbf{C}$ is the fourth-order elasticity tensor and : denotes the 'double-dot product' $\mathbf{S} : \mathbf{T} = S_{ij}T_{ij}$.

The constitutive equations (1) and (2) together define the non-local constitutive behavior of a Hookean solid. Eq. (1) represents the weighted average of the contributions of the strain field of all points $\mathbf{x}'$ in the body to the stress field at point $\mathbf{x}$. In view of the difficulty in using the integral constitutive relation, Eringen [9] proposed an equivalent differential model as

$$(1 - \mu_0^2 \nabla^2)\overline{\boldsymbol{\sigma}} = \boldsymbol{\sigma}, \quad \mu_0 = \tau^2 \ell^2 = e_0^2 a^2 \tag{3}$$

where e_0 is a material constant, and a and ℓ are the internal and external characteristic lengths, respectively. It is assumed that when the local stress tensor is expressed in terms of the displacement gradients through the generalized Hooke's law, the displacements appearing on the right-hand side of Eq. (3) are the non-local displacements.

3. Equations for classical Euler–Bernoulli beam theory

In this section, equations of motion of the Euler–Bernoulli beam theory are derived using the dynamic version of the principle of virtual displacements [54] with the von Kármán non-linearity accounting for moderate rotations. Since the principle of virtual work is independent of the constitutive relations, the