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Dynamic response of continuous beams with discrete viscoelastic supports under sinusoidal loading

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ABSTRACT

Analysis of vibrations of continuous beams with discrete viscoelastic supports has been established through theoretical modeling and a finite element analysis. The theoretical model is based on the Euler–Bernoulli theory, and the Ritz approach was employed to obtain numerical results from which the attenuation of the beam's vibration was obtained. In parallel, a finite element analysis was carried out in ABAQUS using 3D beam elements. It is shown that the results of theoretical calculation agree well with those of the finite element analysis.

Both models were applied to explore geometric and design variations, and then to a full model of a bridge expansion unit as an application example. The vibration of the beams in the design, the influence of the stiffness and the viscous damping coefficient of the supports were discussed, demonstrating the models' usefulness in helping with design optimization.

1. Introduction

Expansion units and joints are commonly used in bridges or viaducts to accommodate temperature-induced movements between bridge decks and abutments. The functions of the units are twofold: they need to limit the internal stresses due to thermal expansion under high temperatures which may cause buckling failure of the long-span beam decks; and to minimize gaps caused by shrinkage under low temperatures for smooth traffic flows over the bridge [1–3]. With the development of motorways, city road viaducts and elevated high speed railway lines, more stringent requirements have been placed on the development of bridge expansion units to allow for large gaps/displacements. Such expansion units are used in hostile environmental conditions and loaded heavily by high volume traffics. The maximum gap between contiguous beams can now reach to 80 mm [4–6]. Fatigue failure under repeated impact is the main cause of damage to bridge/viaduct structures. Apart from the required strength, the expansion units also need to be of low costs, can be installed easily, and require minimum maintenance with long durability.

Fig. 1 shows an example of a module design of a bridge expansion unit. Crossbeams are laid in parallel in the direction of the axis of the bridge underneath the bridge deck and are allowed to expand/shrink freely through the use of sliding bearings. They support a number of beams (here named I-beams, as being used in the design and to differentiate with the crossbeams) crossly laid on top of cross-beams and leveled to the surface of the deck with predetermined equal distance/gaps amongst the I-beams. The number and the size of the I-beams required are determined by the overall deck gap the expansion unit needs to accommodate. And the number of the crossbeams needed depends on the width of the bridge.

With the temperature effect on the crossbeams, the supported I-beams move with the crossbeams to mitigate the gaps amongst them. When designed properly, this mitigation movement will enable the gaps between the I-beams to remain within the required range under all weather conditions to reduce the impact loading caused by the traffic over the gaps.

The dynamic response of the beams (both the I-beams and the crossbeams) needs to be analyzed for potential structural damages [7]. Wang [8] did fatigue tests on several modular expansion units, showing that the residual stress in the beams increases with the number of load cycles. Dexter et al. [9] did study on the structural design, installation and maintenance of expansion joints, suggesting that elastomeric parts and fasteners are best addressed through performance tests on the modular joint unit as a system. Coelho et al. did the dynamic tests of modular expansion joints [10,11]. They showed that the traffic speed has influence on the strain distribution along the center beam, and the design of the modular joint systems must pass both static and dynamic performance tests. Michael [12]

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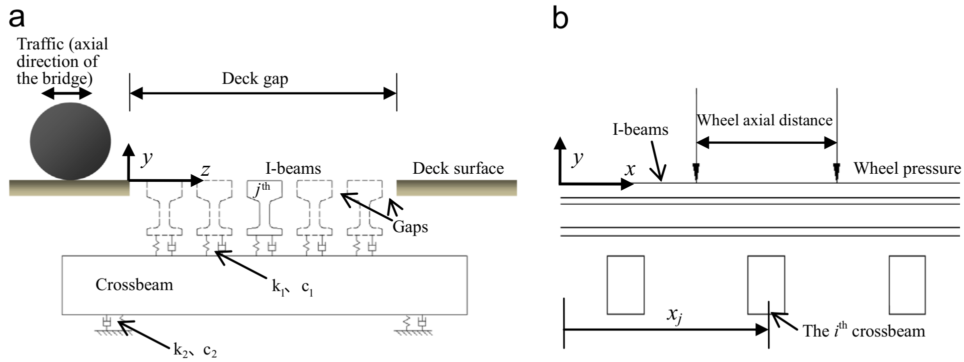

Fig. 1. A modular design of an expansion joint composing of 5 I-beams perpendicular to the traffic supported by a number of crossbeams aligned to the axis of the bridge. (a) Bridge side view and (b) traffic direction view.

established the load form and a theoretical model of the lamella beam-grid expansion joint, and concluded that the dynamic amplification is important for design, and in some cases, its value is higher than those prescribed in the current design codes. Roeder [13] studied the fatigue of modular expansion joints, and showed the importance of the load spectrum on the fatigue life of expansion units. Chaallal O [14] analyzed the results of fatigue tests and provided detailed stress distributions. Crocetti [15] proposed a design load approach through fatigue tests. Ghimire [16] studied the noise generation and radiation from a modular expansion joint. These studies were focused on the performance of whole joint units, where understanding on the responding mechanism of the individual beams and the influence on the viscous supports is weak.

For the purpose of strength and fatigue analysis, the design of the expansion unit can be approximately modeled as continuous elastic beams with discrete viscoelastic supports. This is applicable to both the I-beams on top, and the crossbeams beneath. Most of the studies on beam with elastic foundations are for continuous, non-interrupted supports, such as those in [17–20] where different material models and loading conditions are considered. Yu and his colleagues did dynamic analysis of impact loading on beam-on-foundation based on a simplified rigid-plastic model [17]. Chen and others studied the elastoplastic beam-on-foundation model, mainly on the quasi-static behavior [18,19]. Zhou et al. studied the elastic behavior of ring-on-foundation [20]. However, analyses on beams under discrete viscoelastic supports are rare.

In this paper, we present an analytical solution for the response of beams on discrete viscoelastic supports under dynamic loading. Numerical simulations using finite element code ABAQUS were also obtained and compared with the theoretical model results, through which, the mechanical properties of the system are analyzed for design purpose.

2. Dynamic equations

For continuous beams, transverse vibrational equations can be obtained by the Euler–Bernoulli theory. In Fig. 1, z represents the axial direction of the bridge, i.e. the traffic direction, y the vertical downwards direction and x the direction of the bridge width. Let $y_{Ij}(x,t)$ denote the vertical displacement of the jth I-beam, where the subscript I indicates the I-beam. The vibration equation of the I-beam is given by [21]

$$EI_1 y_{Ij}^{(4)}(x,t)+m_1\ddot{y}_{Ij}(x,t) = -\sum_{i=1}^{NI} F_{cz}(t)\delta(x-x_i)+P(x,t)[\delta(x-x_r)+\delta(x-x_l)] \tag{1}$$

where EI_1 is the flexural rigidity of the I-beam. The superscript (4) represents the 4th order derivative with respect to time, m_1 the mass per unit length, F_{cz} the supporting force by the crossbeams underneath. NI is the total number of supports to the I-beam or the number of the crossbeams in the expansion unit, and x_i the position of the ith support. P is the impact force from the traffic with subscripts r and l representing the right and left wheels of the vehicle. δ is the Kirchhoff function.

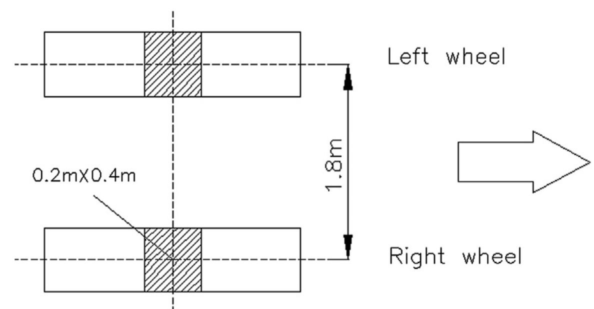

Fig. 2. Geometrical description of wheel contact.

$$\delta(x-x_i) = \begin{cases} 0 & x \neq x_i \\ 1 & x = x_i \end{cases} \tag{2}$$

In modern bridge design [1,16], the standard axle load is considered 140 kN up to a velocity of 100 km/h. The contact area of a wheel and the road surface is assumed to be 0.2 m by 0.4 m, and the axial distance between the two wheels is taken as 1.8 m, as shown in Fig. 2. For the most unfavorable condition, the wheels are assumed loading at the mid span between neighboring supports as shown in Fig. 1(b). The dynamic load pulse of a wheel can be described by a sine wave as shown in Fig. 3(a) [12]. The time period of the half wave depends on the traffic speed.

Let $P(x, t)$ be the pulse loading of one wheel, we assume

$$P(x,t) = -\frac{P_n}{2}[1+\sin(\omega t-\pi/2)] \tag{3}$$

where P_n is the weight loading of a vehicle and remains constant. The velocity dependent impact effect will be considered later by an impact coefficient (in Section 4.3).

For the supporting forces received by the I-beam, we can obtain.

$$-\sum_{i=1}^{NI} F_{cz}(t)\delta(x-x_i) = \sum_{i=1}^{NI} k_1[y_{ci}(x_j,t)-y_{Ij}(x_i,t)]+\sum_{i=1}^{NI} c_1[\dot{y}_{ci}(x_j,t)-\dot{y}_{Ij}(x_i,t)] \tag{4}$$

where k_1 and c_1 are the spring stiffness and viscosity coefficient of the contact between the I-beam and the crossbeam.