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Theory of ice-skating ☆

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ABSTRACT

Almost frictionless skating on ice relies on a thin layer of melted water insulating mechanically the blade of the skate from ice. Using the basic equations of fluid mechanics and Stefan law, we derive a set of two coupled equations for the thickness of the film and the length of contact, a length scale which cannot be taken as its value at rest. The analytical study of these equations allows to define a small a-dimensional parameter depending on the longitudinal coordinate which can be neglected everywhere except close to the contact points at the front and the end of the blade, where a boundary layer solution is given. This solution provides without any calculation the order of magnitude of the film thickness, and its dependence with respect to external parameters like the velocity and mass of the skater and the radius of profile and bite angle of the blade, in good agreement with the numerical study. Moreover this solution also shows that a lubricating water layer of macroscopic thickness always exists for standard values of ice skating data, contrary to what happens in the case of cavitation of droplets due to thermal heating (Leidenfrost effect).

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1. Introduction

Skating is possible because of the very small friction felt by a thin blade sliding on ice, in sharp contrast with much larger solid-on-solid friction observed at temperatures well below the melting temperature. This has long been explained [2] by the existence of a thin lubricating layer of liquid water. Standard wisdom was that ice melts because of the local increase of pressure by the weight of the skater, a pressure leading to local melting because of the pressure dependence of equilibrium melting temperature. But this melting is an equilibrium phenomenon, then it is unclear how it can describe the continuous formation of liquid water from ice, in particular because this requires a supply of heat. According to another explanation [3] the main phenomenon is melting by the heat generated by friction in-between ice and the sliding skate. This heat yields the energy needed to balance the latent heat, as given by Stefan condition on the ice surface. A condition of mechanical equilibrium expresses also how the weight of the skater is supported by the pressure generated inside the film, which avoids direct ice-skate contact.

We reconsider below this question in the light of a macroscopic approach, assuming that the liquid layer in-between ice and skate has macroscopic thickness. Even though this layer is often referred to as a lubricating layer, it acts differently of a regular lubricant: lubricants like oil are chemically different of the solids facing each other. Therefore a layer of molecular thickness can change the conditions of sliding. On the contrary, in the present case, the lubricating layer is made of the same molecules of water as solid ice, so that a layer of molecular thickness, which is about 1 Å for water, will be mixed with the molecules present at the surface of ice, and cannot change the ice-skate friction.

Our analysis is inspired by previous work on the Leidenfrost effect [4] where a thin layer of vapor was also analyzed and where the balance of vertical forces (weight of the airborne Leidenfrost drop and pressure forces) plays a central role. A difference between the skating problem and the Leidenfrost effect is that in Leidenfrost the evaporation from the droplet (instead of the melting in skating) is due to the heating by the hot bottom plate although in skating

☆ *Foreword written by Yves Pomeau*: The present paper reports original work which seems to me a very appropriate topic for this volume honoring Martine Ben Amar. Its matter is related to my first (very successful) collaboration with her. Our first joint paper [1] studied the growth of needle crystals in an undercooled melt. At the time it was well understood that the classical Ivantsov solution for the needle crystal had to be supplemented by physical effects outside of the ones in Ivantsov theory to yield a solution which would be both unique and pertinent for the observations. It had long been suspected that surface tension, through the Gibbs–Thomson curvature-dependence of the equilibrium temperature, had to be taken into account. But, at the time, it was more or less an article of faith. With Martine, we did show that things did not work as believed by many in the field: no solution exists with isotropic solid-melt surface tension. Crystal anisotropy had to be taken into account [1]. This advance was followed by many others thanks to Martine and her collaborators so that this tricky problem can now be considered as solved.

The connection with the present work is that, as in the 1986 Europhysics Letter paper, we have to introduce Stefan condition for melting to understand quantitatively how a supply of heat can explain the dynamics of the solid/liquid interface. Ice skating relies on the same dynamics, and as the literature shows too well, it is a quite non-trivial matter to write the appropriate boundary conditions. It is quite amazing to come back to this "classical", but somewhat forgotten, physics after so many years. I wish you Martine, many more years of active and fruitful research.

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the heat comes from viscous friction in the bulk of the lubricating layer. Here it is the longitudinal viscous friction (along the direction of motion) which is responsible for heating the film of water, although this longitudinal flow does not generate any vertical stress in the layer. The weight of the skater is actually supported by the transverse Stefan flow which undergoes a drop of pressure bigger than in the longitudinal direction in the skating case (this is not the case treated in [3], which considers a 2-D situation).

We answer a very natural question: when a sharp edge slides on ice, how deep is the furrow due to melting arising itself from the heat generated by viscous friction? This introduces a length scale (the depth of the furrow) which turns out to be much bigger than the thickness of the liquid layer due to melting, a layer in-between ice and skate. To estimate this contribution of melting one needs to find out how much water is made during the time a skate passes over a point on the ice surface, and so how much ice has melt to make the furrow. We show that this “lubricating” layer has macroscopic thickness in concrete situations, but that it is far thinner than the depth of the furrow, an important remark. Furthermore, as written above, the depth of the furrow is far smaller than its length which is far smaller than the radius of profile of the blade. Hence one has to solve a problem with four different length scales, each one with a different order of magnitude than the three others, see Eq. (1) referring to the lengths reported in Fig. 1.

Here we present a theory using in a consistent way Stefan description of the melt/solid interface. This is not a trivial endeavor, because one has to write the equations for fluid mechanics in the frame of reference where the boundaries of this film are fixed, namely in the frame of reference of the skater. Therefore when writing Stefan condition on the surface of ice, we must consider the interface as a moving surface with respect to ice *and* the ice as moving with respect to the skater. It follows that the ice melting rate cannot be identified to the growth rate of the film thickness, as done in the recent model derived by Lozowski et al. [5,6]. We show below that the missing term in their model is actually of prime importance.

By combining (i) Stephan condition with (ii) the balance between the weight of the skater and the viscous pressure force in the liquid layer, and (iii) the relation between the volume of the trough and the melting process during the passage of the skater, we derive two coupled integro-differential equations relating the film thickness to the length of contact. All physical lengths of the problem are then deduced from the data, in particular the length of contact between ice and skate, a parameter which cannot be set to its value at rest as done in [5,6]. The study of these coupled equations allows to define a small a-dimensional quantity depending on the longitudinal coordinate (in the direction of the motion) which can be neglected everywhere except close to the contact points at the front and the end of the blade. Help to this property we show that the length of contact can be deduced from the integral equation with a very good approximation. Then it remains to solve a single differential equation for the film thickness. From the analytical study of the two boundary layers at the front and at the end of the blade, we show that one can deduce an expression

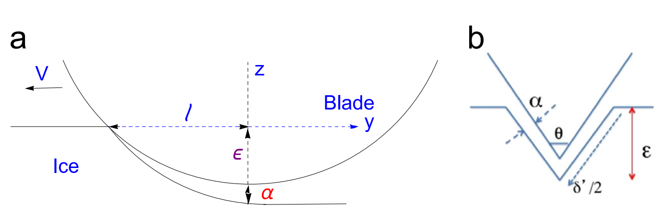

Fig. 1. Schema of the blade digging into the ice surface. In (a) we set $y=0$ at the front contact point and $y=\ell$ at the back contact point where the letters ϵ and α are placed. (b) Schema of the cross-section at a given value of y. The geometry is deformed in order to see the various lengths.

of the order of magnitude of the film thickness in these two regions in terms of the data. This provides without any calculation the relation between the unknown variables and the external parameters like the velocity and mass of the skater and the radius of profile and bite angle of the blade, in good agreement with the numerical study.

In Section 2 we derive the model equations for standard ice-skating conditions, namely for V-shaped blade transverse profile, and study in details the solution analytically and numerically with applications to hockey and inclined speed- skating blades. Because of the existence of a small parameter we define a critical mass below which no skating is possible. This critical mass is so small that one may conclude that a macroscopical film of water is always formed in the skating case, a conclusion not valid for cavitation of water droplet in the Leidenfrost effect [4]. The case of a vertical speed-skating or rectangular transverse blade profile is treated in Section 3 which gives results very similar to the ones for V-shaped blades, although with more complex analytical expressions. Last section is devoted to conclusions.

2. Equations for V-shaped blades

To get close to ice-skating conditions, we consider first a skate having a smooth surface ending toward the ice with a sharp edge, its longer dimension being in the direction y of the imposed speed, see Fig. 1. Such V-shaped blades are the ones sharpened for hockey skaters and also figure skatings. The bottom of each blade presents actually a hollow separating two V-shaped edges but the skater spends most of his time on a single edge, therefore we focus here on the case of V-shaped transverse profile on contact with the ice surface via a thin film of melted water. In this geometry the water layer has only a finite extend along the horizontal coordinate x perpendicular to the direction of motion, a width much smaller than the length of the skate. Lastly the vertical direction is associated to coordinate z. The curved profile of the blade and the cross-section are schematized in Fig. 1 where the length scales are not respected. The length of contact ℓ is defined as usual, as the longitudinal dimension of the wetted part of the blade which is situated between the first point of contact (where the thickness of the layer is zero) and the point of maximum penetration of the blade into the ice. Beyond this point the contact is assumed to stop because the bottom of the curved blade becomes above the surface of the through dug by the passage of the skater. This geometry also describes the case of speed skating (having rectangular cross-section) with inclined blade making an angle $\pi/4$ with the vertical axis, considered in Section 2.3.

We neglect the fact that sharpening of the blade leaves a slightly rounded cylindrical wedge with a very small radius. This is a reasonable assumption because the water layer has a thickness of order of fifteen micrometers, that is much larger than the radius of sharpening (about one micrometer). Therefore we consider a perfectly sharpened blade with V-shaped wedge. This hypothesis may be invalid in a very small domain (in front of the skate where the liquid layer is very shallow), that is not essential for our purpose which is to derive the geometry of the furrow whose main part has length scales much larger than the radius of curvature or the imperfections of sharpened wedges.

We derive below the equations for the flow created by the friction forces along y which is partially evacuated laterally by a secondary flow flowing in the x direction (perpendicular to the plane of Fig. 1(a) generating a pressure supporting the weight of the skater. Note that there is also some flow ejecting fluid behind the blade which is generated by the pressure gradient along y. This gradient is negligible with respect to the lateral one because the

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