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Faraday instability in floating drops out of equilibrium: Motion and self-propulsion from wave radiation stress

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ABSTRACT

Instabilities in fluids are usually studied in domains with fixed boundaries or free to grow in space. In the Faraday instability, a liquid undergoing vertical oscillation is unstable to surface waves. Recently, a novel situation has been explored in which Faraday waves are triggered in floating drops that behave as deformable domains. A mutual adaptation between wave pattern and drop shape occurs, that leads either to equilibrium or out-of-equilibrium behaviors depending on the magnitude of wave radiation pressure over capillary pressure at the drop boundary. Here we investigate experimentally the out-of-equilibrium system, in which the radiation pressure exceeds the capillary response. The drop is abruptly deformed by the waves and possibly splits in fragments that have complex dynamics. These dynamics are explained by the radiation stress exerted by Faraday waves at boundaries. In particular, a simple model is able to predict the limit speed of self-propulsion of croissant-shaped drops.

1. Introduction

In 1831, Michael Faraday discovered that a liquid undergoing vertical vibration is unstable to waves [1]. In this parametric instability, gravity is modulated periodically and patterns of standing waves appear on the liquid surface. Waves oscillate at half the forcing frequency and the instability threshold amplitude increases with liquid viscosity. Benjamin and Ursell developed the linear theory of the instability in terms of surface eigenmodes and showed that their excitation is ruled by the Mathieu equation [2]. Kumar and Tuckerman developed a complete Floquet analysis of the full hydrodynamic problem and accounted for the observed instability threshold [3]. Muller et al. performed a systematic perturbative treatment of the linear stability to investigate the relation of the instability threshold with resonant cavity modes [4]. Douady investigated experimentally Faraday instability patterns in several simple geometries, showing the relation with cavity modes [5].

Faraday instability patterns are only one example of hydrodynamic instability patterns which depend on the geometry of the container in which they develop. A notorious example is the Rayleigh–Bénard instability, where the spacing between hot and cold plate fixes the typical size of the convection rolls formed by the fluid [6,7]. In the opposite situation instabilities are free to grow in space, as the expansion of thermal plumes above a heat source [8,9]. The Saffman–Taylor instability [10] occurs both in confined and free geometries. In open circular geometries it results in fractal structures, while a thickness gradient leads to the periodic fingers of the printer instability [11].

Recently, an intermediate situation has been investigated, in which the Faraday instability is triggered in deformable domains having flexible boundaries. Liquid drops have been deposited on a viscous liquid bath, the two liquids being immiscible. In the absence of forcing, drops have a circular contour in the horizontal plane, while the vertical profile depends on their size. The profile of small drops is fixed by the Young–Laplace laws of capillarity at the triple line, while for large drops it results from a balance between hydrostatic pressure and capillary pressure. Large drops have a pancake shape with roughly constant thickness and meniscus at the border [12–14]. When the Faraday instability is triggered, drops behave as deformable domains and mutual adaptation between wave pattern and domain shape is observed [15]. Faraday waves exert a radiation pressure on the drop boundary, whose response is given by the capillary pressure. Accurate exploration of different liquid pairs lead to two archetypes of behavior, depending on the magnitude of the wave radiation pressure P_r over the capillary pressure P_L (Laplace pressure):

$$a_0 = \frac{P_r}{P_L} \quad (1)$$

In the first archetype, the radiation pressure is balanced by the capillary pressure and $a_0 \sim 1$. The mutual adaptation results in a

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steady shape that was investigated experimentally and found theoretically as a solution of a Riccati equation accounting for the effect of both pressures [16]. In the second archetype, the radiation pressure exceeds the response of the drop boundary and $a_0 \sim 100$. The system is out of equilibrium, the initial drop stretches and possibly breaks in fragments that can be steady or having complex dynamics [15,17].

In this paper we develop the experimental investigation of the Faraday instability in drops floating on a viscous bath in the out-of-equilibrium system. We report observations of various shapes obtained after the fragmentation of the initial drop and study their dynamics. We show that the dynamics can be explained by the radiation stress exerted by Faraday waves at the drop extremities. In particular, we predict the limit speed of croissant-shaped fragments which undergo self-propulsion.

2. Experimental setup

The experimental setup is shown in Fig. 1. A circular cell (radius 10 cm, depth 0.8 cm) is fixed to a shaker which forces it to oscillate vertically with tunable frequency and amplitude. The effective acceleration $\gamma(t) = \gamma_m \cos 2\pi f_0 t$ of the cell is measured via an accelerometer vertically fixed on its wall. In the experiments, frequency and amplitude ranges respectively from 50 Hz to 200 Hz and from 0 to 10g. The signal is acquired by an oscilloscope through a signal conditioner. The error on the acceleration amplitude is 0.05g, while on frequency it is lower than 0.1 Hz.

The cell is first filled with viscous silicon oil, having density $\rho_2 = 965$ kg/m, surface tension $\sigma_2 = 20.3 \pm 0.3$ mN/m and dynamic viscosity $\mu_2 = 100$ mPa s, until the bath depth $H = 5.0$ mm is reached. A drop of ethanol is then deposited on it, having $\rho_1 = 789$ kg/m, $\sigma_1 = 22.8$ mN/m, $\mu_1 = 0.9$ mPa s. The two liquids are immiscible and the drop floats on the liquid bath. The interfacial tension has been measured via the pendant drop method and it is $\sigma_{12} = 0.7 \pm 0.1$ mN/m. When we deposit the drop we observe that it does not spread, and calculation of the wetting parameter [14] leads to the negative value $\Sigma_{12} = -2.2 \pm 0.5$ mN/m. Moreover we observe that the drop is covered by oil and this is in agreement with the positive value of the wetting parameter of oil on ethanol $\Sigma_{21} = 1.8 \pm 0.5$ mN/m. As the instability threshold increases with liquid viscosity, the strong viscosity contrast allows for triggering the Faraday instability in the drop while the bath is largely below its Faraday threshold.

Photographs and videos of the system are acquired in two different configurations (Fig. 1):

- top views to observe the horizontal shape of the drops and measure the Faraday wavelength (a).
- Side views to measure thickness and wave amplitudes (b).

In top view configuration the videocamera is fixed vertically above the cell (Fig. 1(a)). Horizontal zones of the liquid surface reflect light beams towards the videocamera, so that white regions on the images correspond either to crests or troughs of the waves. The Faraday wavelength is therefore measured as the distance between two crests or troughs.

In side view configuration the videocamera is fixed in front of the cell while the light source is behind. Transparent plexiglass cells are used to visualize both the emerged and submerged parts of the drop. The bath surface is pinned onto a razor blade stuck at the edge of a wall in order to eliminate the meniscus the liquid would form with the wall (Fig. 1(b)). The wave amplitude $\zeta_0 \sim 0.5$ mm is measured by strobing the oscillation close to the Faraday frequency and using large magnification.

3. Experimental findings

We investigate experimentally the behavior of a large drop of volume 1 ml undergoing the Faraday instability (Fig. 2). We fix the forcing frequency at $f_0 = 100$ Hz and increase progressively the

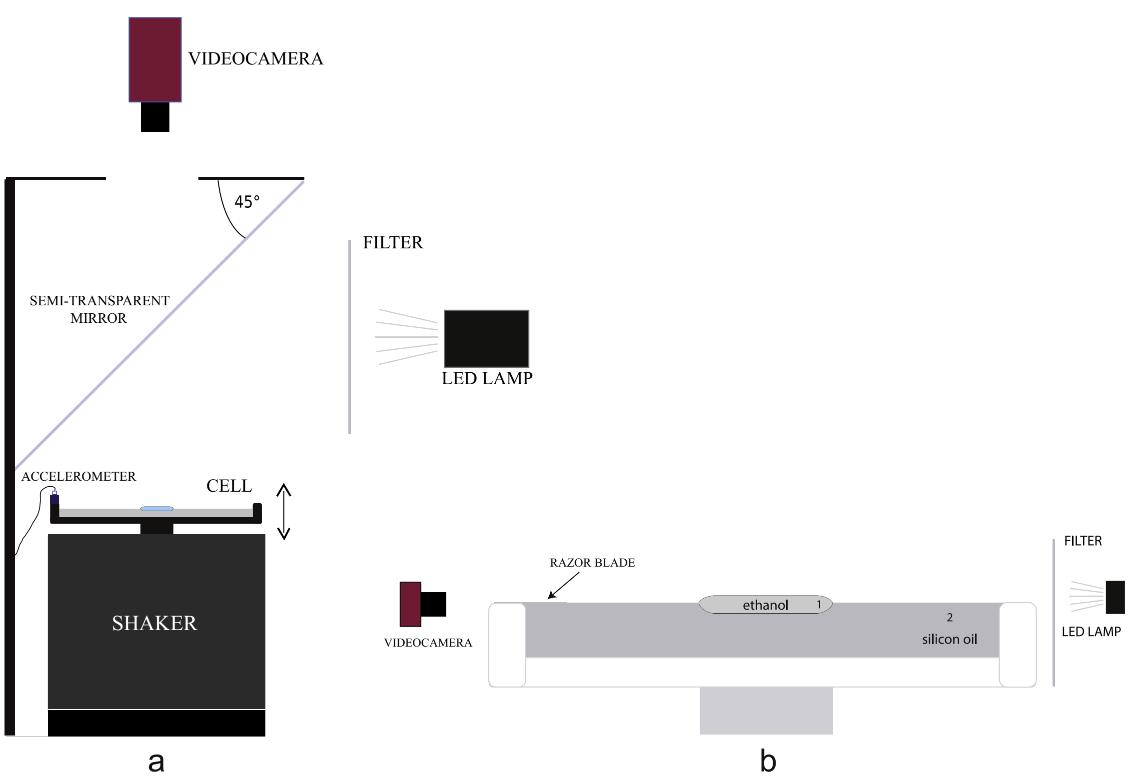

Fig. 1. Experimental setup for top-view video acquisition (a) and detail of the side-view configuration (b).