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Elastic–plastic stress investigation for an arc-shaped interface crack in composite material

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ABSTRACT

The plastic zone size and crack tip opening displacement have been investigated for a curved interface crack between a circular inclusion and an infinite matrix. The mixed-mode Dugdale model is used to study the plastic deformation where the stresses in the plastic zones satisfy the Von Mises yield criterion. The plastic zone size at the crack tip is calculated by satisfying the condition that the complex stress intensity factors induced by external load and those induced by closure stress cancel off. With the distributed dislocation method, the physical problem is formulated into a set of singular integral equations which are numerically solved by using Jacobi polynomials. The influence of the material properties and other geometric parameters on the stress intensity factors (SIF), plastic zone size (PZS) and crack tip opening displacement (CTOD) is discussed in detail. The numerical examples show that both the crack debonding angle and the inclusion/matrix shear modulus ratio have significant influence on the normalized values of SIF, PZS and CTOD. The normalized PZS reaches its maximum value when the crack debonding angle is 90°. The effect of shear modulus ratio is very significant when the inclusion is "softer" than the matrix. When the inclusion is much "stiffer" than the matrix, the inclusion plays a dominant role. Changing the shear modulus ratio does not have great influence on the plastic deformation ahead the crack tips.

1. Introduction

Composite materials are engineered or naturally occurring materials made from two or more constituent materials with many unique and superior properties. Composites have widely been used in diverse engineering applications, from circuit boards to surfboards to insulation for space shuttles. As defects such as micro-cracks or debonding are often observed at the interface of dissimilar constituents in composites, great attention has been paid to understand the fracture behavior of these materials with interface cracks. The analysis of straight interface cracks between dissimilar materials was pioneered by Williams [1], then extended by Rice and Sih [2] in the early development of the subject.

However, for fiber-reinforced composites or any composite material with circular inclusions, cracks along the interface of fiber/matrix or inclusion/matrix due to debonding are curved (arc-shaped). The growing importance of composites and alloys has focused more and more attention on arc-shaped cracks along interfaces. Stippes et al. [3] reduced the problem of curved cracks to a singular integro-differential equation which can be solved exactly when the crack is in a single homogeneous material. Wilson [4] extended this work and solved the integro-differential equation approximately for dissimilar materials. England [5], Perlman–Sih [6] and Toya [7] investigated the problem of a crack lying on the interface between a circular inclusion and an infinite plate, by reducing the problem to the solution of Hilbert problem with the complex function theory. Kelly et al. [8] presented the distributed dislocation method to formulate the plane problem of a crack between a circular elastic inclusion and an elastically dissimilar matrix, and some special cases solved by the Gauss–Chebyschev quadrature were given as examples. Xiao [9] carried out the stress investigation for the problem of a penny-shaped crack located above the pole of a spherical particle (inhomogeneity) in 3D elastic solid under tension. Fan and Xiao [10] studied a Zener–Stroh crack near an interface, and found the stress intensity factor and the critical crack length are strongly affected by the presence of the interface. Theotokoglou et al. [11] gave the analytical solution for a matrix containing a single partially debonded circular inclusion with non-uniform far field loading. The obtained solution was applied to study the problem of the interface crack (formed by debonding) interacting with a crack in the matrix. Prasad and Simha [12] examined the problem of an interface arc crack around a circular elastic inclusion embedded in an elastic matrix, and demonstrated a general approach for

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generating uniform/non-uniform stress fields inside a test region. Gorbatikh et al. [13] proposed a method to estimate the effect of partially debonding at matrix/inclusion interfaces on the overall elastic compliance, studying the energy released during the debonding process. Using the boundary element method, Paris et al. [14] developed a micromechanical model to investigate the growing and kinking of an interface crack between fiber and matrix under loading transversal to the fiber. Mantic [15] proposed a theoretical model for the simultaneous prediction of the initial size of a crack originated at the inclusion/matrix interface and of the critical remote tension required to originate this crack. Luo and Xiao [16] investigated the interaction between a screw dislocation and an elliptical nano inhomogeneity embedded in an infinite matrix, and found the interface stress effects of the nano inhomogeneity are accounted for with the Gurtin–Murdoch model. Recently, an analytical method has been developed by Kushch et al. [17,18], and an exact solution has been obtained on the elasticity problem for a plane containing a finite array of partially debonded circular inclusions. Additional research work on fiber-reinforced composites can be found in Refs. [19–21].

Although a lot of research work has been done on curved interface crack, there is still a lack of literature for these problems with plastic zone corrections at the crack tips. For ductile materials (such as metal-matrix composites), fracture analyses could be more accurate if plastic zone corrections at crack tips were made, and the crack tip opening displacement criterion seems to be more applicable to judging if a fracture will take place. Particularly for interface crack problems, it is not practical for engineers to use complex stress intensity factors as parameters to judge if a fracture will occur.

In our current work, an approach to solving the plastic zone size (PZS) and crack tip opening displacement (CTOD) for arc-shaped interface cracks is developed based on the mixed-mode Dugdale model [22]. The PZS and CTOD are evaluated for an interface crack between a circular inclusion and an infinite matrix. To the best knowledge of the authors, it is the first time the elastic–plastic fracture behavior of a curved interfacial crack is evaluated. The mixed-mode Dugdale model is developed to deal with the plastic deformation, where the Von Mises yield criterion is applied on the plastic zones. The PZS is calculated by satisfying the condition that the complex stress intensity factors vanish, which means the stress intensity factor induced by the external load must be equal and opposite to that induced by closure yielding stress in the plastic zone. The relative CTOD can be gained by dislocation theories after the PZS is obtained.

In Section 2, the mixed-mode Dugdale model is introduced to solve the PZS and CTOD ahead each crack tip. In Section 3, numerical examples for the curved interface crack under remote uniform tensile and hydrostatic loading are given. The effects of the half debonding angle ω and the shear modulus μ_2/μ_1 ratio on the normalized values of PZS and CTOD are studied and discussed. Some conclusions are drawn in Section 4. The detailed mathematical formulations of the physical problem is given in the Appendix.

2. The physical problem and formulation

2.1. The arc-shaped interfacial crack with plastic zone corrections

The physical problem is shown in Fig. 1(a), where the circular inclusion is embedded in an infinite matrix or the matrix is sufficiently large in comparison with the inclusion (with radius r_0). There is an arc-shaped interface crack over the region $r=r_0$, $-\omega \le \theta \le \omega$. At the crack tip of each side, there is a plastic strip over the region of $\omega < |\theta| < \omega+\rho$ along the interface. The mechanical properties of the matrix and the inclusion are denoted by “1” and “2”, respectively. The normal stress σ_{ys_rr} and the shear stress $\sigma_{ys\_r\theta}$ in the plastic zone at the right crack tip are shown in Fig. 1 (b). The parameter δ is used to denote the crack tip opening displacement.

The normal stress and the shear stress in the plastic zone should satisfy the Von Mises yield criterion and have the following form:

$$\sigma_{ys}^2 = \sigma_{ys\_rr}^2 + 3\sigma_{ys\_r\theta}^2. \tag{1}$$

Here, σ_{ys} is the lower yielding stress of the two materials. When the materials are given, the yielding stress is a constant.

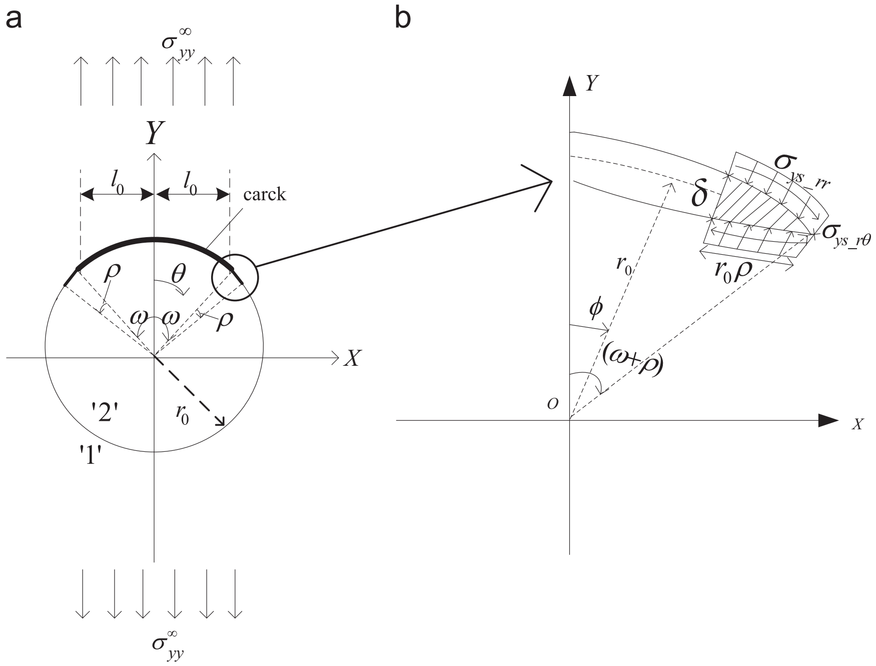

Fig. 1. An arc-shaped interface crack between an inclusion and an infinite matrix under remote tension loading. (a) An arc-shaped interface crack with the plastic zone size (arc angle) ρ at each crack tip; (b) the normal and shear stresses in the plastic zone.