International Journal of Mechanical Sciences 83 (2014) 124–132

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Free vibration of moderately thick antisymmetric laminated annular sector plates with elastic edge constraints

Ashish Sharma *

Department of Mechanical Engineering, DIT University, Dehradun 248 009, India

ARTICLE INFO

Article history:
Received 28 August 2013
Received in revised form
18 March 2014
Accepted 1 April 2014
Available online 13 April 2014

Keywords:
Free vibration
Mindlin sector plates
Antisymmetric
Elastic edges
DQM

ABSTRACT

This study presents, for the first time, free vibration results of laminated sector plates having elastic edges. Each layer is cylindrically orthotropic. A differential quadrature method (DQM) based formulation is used for studying the problem of shear-deformable laminated sector plates having translational as well as rotational edge constraints. By varying the edge stiffnesses, several combinations of simply supported and clamped edge conditions are simulated. The results are further validated by comparison with the results of laminated square plates (with elastic edges) and of laminated sector plates (without elastic edges) available in the literature.

1. Introduction

Laminated plates can be made use of in several engineering applications like aeronautics, space, automobiles, bridges. In comparison to rectangular plates, the literature corresponding to sector plates is relatively scant. Bending, buckling and vibration problems of sector plates of different levels of computational complexity have been the subject of many research papers [1–12].

Traditionally, as well as recently, these papers have been presenting results pertaining to various combinations of the so-called clamped, simply supported and free edges. Most of the research, including the most recent ones [6,7,12], have been carried out using such classical boundary conditions. In these classical boundary conditions, corresponding to every degree of freedom, either the corresponding force (natural boundary conditions) or the displacement (essential boundary condition) is prescribed [13]. The more realistic boundary conditions have been found to be the ones which involves some suitable relationships between the displacement components and the corresponding forces. Such edge conditions have been investigated for rectangular [14–22] using a model of edge conditions being termed as 'elastic edges.' For sector plates having elastic edges, such literature has been rather scant [2,3].

For example, Sharma et al. [6] had carried out free vibration and buckling study of sector plates having various combinations of free, clamped and simply supported edge conditions. They had employed two-dimensional Chebyshev polynomials for the spatial discretization. Zhou et al. [7] employed Chebyshev–Ritz method for 3-dimensional free vibration analysis of sector plates having clamped, simply supported and free edge conditions. Aghdam et al. [12] presented the extended Kantorovich method for static analysis of moderately thick functionally graded sector plates having fully clamped edges.

With respect to rectangular plates having elastic edge conditions, several contributions have been made to the available literature. Liew et al. [14] presented the first known results, apparently, of free vibration analysis of symmetrically laminated cross-ply rectangular plates with edges having uniform elastic restraints – translational as well as rotational. Gorman [15] used the superposition-Galerkin method and Zhou [17] applied the Rayleigh–Ritz method along with static Timoshenko beam functions for computing the natural frequencies of isotropic Mindlin rectangular plates. Shu and Wang [16] presented the vibration analysis of thin isotropic plates with mixed and nonuniform boundary conditions using generalized differential quadrature method. Ashour [18] did the vibration analysis of variable thickness isotropic plates with edges elastically restrained against both rotation and translation using the finite strip transition matrix technique. Karami et al. [19] studied the natural frequencies of moderately thick symmetrically laminated rectangular plates with elastically restrained edges using the differential quadrature method (DQM). Ohya et al. [20] presented an interesting study of the free vibration of rectangular isotropic Mindlin plates having internal column supports and uniform elastic edge supports. They

* Tel.: +91 95570 95696.
E-mail address: ashishsharma71@gmail.com

http://dx.doi.org/10.1016/j.ijmecsci.2014.04.005
0020-7403/

employed the superposition method while achieving compatibility between the plate and the column by requiring that the column and plate rotations be equal.

Using one and two dimensional Fourier series expansions for the implicit spatial discretization, Li et al. [21] presented an exact series solution for the transverse vibration of isotropic thin rectangular plates with edges having linear as well as rotational stiffnesses. Li and Yu [22] developed an empirical formula based on the analytical results obtained from a combination of Rayleigh–Ritz and finite element methods for predicting natural frequencies of a thin orthotropic rectangular plate with uniformly restrained edges. Sharma et al. [23] applied the DQM for generating free vibration results of antisymmetric cross-ply laminated rectangular plates with elastic edge constraints.

Among the methods used to study such types of problems, DQM is increasingly being used to study the problems whose mathematical model is a set of differential equation(s) – linear or nonlinear, ordinary or partial [24–28]. Shu and Richards [24] applied the generalized differential quadrature method to solve two-dimensional incompressible Navier–Stokes equations. Shu [28] presents a good study of the differential quadrature technique and its various applications to different engineering problems like the ones of Navier–Stokes equation, structural analysis and chemical engineering.

In this paper an attempt is made to extend the free vibration results of sector plates given in [6] to the work given by Karami et al. [19] and Sharma et al. [23] for rectangular plates. This work, thus, aims to study the free vibration problem of symmetrically as well as antisymmetrically laminated sector plates having elastic edge constraints.

Based on Mindlin plate theory, the governing equations are five partial differential equations in cylindrical coordinates. Spatial discretization of these equations is done using the well established DQM. The developed formulation is validated by extensive convergence and comparison studies. The variation of natural frequencies with the variation of edge stiffnesses is studied with respect to the other important geometrical and material parameters like thickness ratio, annularity (r_i/r_o), sector angle, moduli ratio and lamination schemes.

2. Mathematical formulation

Fig. 1 shows the geometry of the sector plate. Each lamina is considered to be cylindrically orthotropic with the fiber orientation being either in the radial or circumferential direction. The layers are assumed to be perfectly bonded. Considering the first order shear deformation theory, the displacement fields are

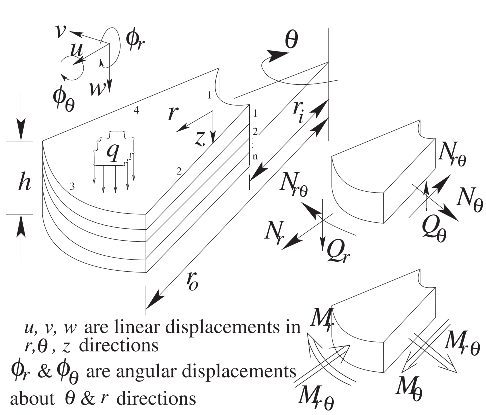

Fig. 1. Geometry of the problem.

expressed as follows [30,6]:

$$u^*(r,\theta,z,t) = u(r,\theta,t) + z\phi_r(r,\theta,t) \tag{1a}$$

$$v^*(r,\theta,z,t) = v(r,\theta,t) + z\phi_\theta(r,\theta,t) \tag{1b}$$

$$w^*(r,\theta,z,t) = w(r,\theta,t) \tag{1c}$$

Here, (u,v,w) are linear displacements of a typical point on the mid-plane $(z=0)$ and (ϕ_r,ϕ_θ) are the rotations of a normal to the mid-plane about respective directions. For simplicity of expression, the following quantities are introduced:

$$r = (r^* - r_i)/(r_o - r_i) \tag{2a}$$

$$\theta = \theta^*/\Theta \tag{2b}$$

$$\mu = r_i/r_o \tag{2c}$$

$$\zeta = \{r(1-\mu)+\mu\} \tag{2d}$$

$$\zeta' = \zeta_{,r} \tag{2e}$$

so that the usual dimensional cylindrical coordinates (r^*,θ^*) are replaced by their non-dimensional counterparts (r,θ), with Θ being the total included angle of the sector plate under consideration. It may be noted that r varies from 0 to 1 from the inner circumferential edge to the outer circumferential edge while r^* varies from r_i to r_o. Further, $((\cdot)_{,r},(\cdot)_{,\theta},(\cdot)_{,t})$ denote differentiation with respect to (r,θ,t).

With the displacement field considered here, $\epsilon_{zz} = \partial w/\partial z = 0$. Other strain displacement relations can be expressed as follows:

In-plane strains at the mid-plane are

$$\epsilon_r^{(0)} = \frac{1}{r_0}\frac{u_{,r}}{\zeta'} \tag{3a}$$

$$\epsilon_\theta^{(0)} = \frac{1}{r_0\zeta}\left(u + \frac{v_{,\theta}}{\Theta}\right) \tag{3b}$$

$$\gamma_{r\theta}^{(0)} = \frac{1}{r_0\zeta}\frac{u_{,\theta}}{\Theta} + \frac{1}{r_0}\frac{v_{,r}}{\zeta'} - \frac{v}{r_0\zeta} \tag{3c}$$

The curvatures of mid-plane are

$$\kappa_r^{(0)} = \frac{1}{r_0}\frac{\phi_{r,r}}{\zeta'} \tag{4a}$$

$$\kappa_\theta^{(0)} = \frac{1}{r_0\zeta}\left(\phi_r + \frac{\phi_{\theta,\theta}}{\Theta}\right) \tag{4b}$$

$$\kappa_{r\theta}^{(0)} = \frac{1}{r_0\zeta}\frac{\phi_{r,\theta}}{\Theta} + \frac{1}{r_0}\frac{\phi_{\theta,r}}{\zeta'} - \frac{\phi_\theta}{r_0\zeta} \tag{4c}$$

The shear strains in rz and θz planes are

$$\gamma_{rz} = \phi_r + \frac{1}{r_0}\frac{w_{,r}}{\zeta'} \tag{5a}$$

$$\gamma_{\theta z} = \phi_\theta + \frac{1}{r_0\zeta}\frac{w_{,\theta}}{\Theta} \tag{5b}$$

The strain components at any point can thus be expressed as

$$\begin{Bmatrix} \epsilon_r \\ \epsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} = \begin{Bmatrix} \epsilon_r^{(0)} \\ \epsilon_\theta^{(0)} \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \kappa_r^{(0)} \\ \kappa_\theta^{(0)} \\ 0 \\ 0 \\ \kappa_{r\theta}^{(0)} \end{Bmatrix} \tag{6}$$

In this study, fiber orientation in a composite laminate (polar orthotropic) is either radial (R) or circumferential (C) so that the material axes coincide with the geometrical axes. The stress–strain