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On the transient response of viscoelastic beams and plates on viscoelastic medium

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ABSTRACT

In the present study, free vibration of isotropic viscoelastic beams and plates on viscoelastic medium is investigated. The Boltzmann superposition integral model with Dynamic Mechanical Analysis (DMA) results is considered for viscoelastic material of the beam and plate to describe more accurately the behavior of material rather than simple complex constants model. The viscoelastic beam and plate on viscoelastic medium modeling are used in railroad vibration with the damping layer on the lateral surface of the rail and the polymeric flooring on the floor as a viscoelastic medium for the beam and the plate, respectively. Weighted residual method and QZ algorithm are applied to obtain beam and plate natural frequency. In addition, medium properties' effects on the natural frequencies of the beam and plate are discussed. For the first time, critical damping of viscoelastic beam is presented due to the greater effect of medium viscosity at all boundary conditions. The unknown coefficients of dynamic response function are calculated by Fourier transform. Closed form expressions are obtained for lateral displacement of the beam and plate with various boundary conditions in terms of complex natural frequency, mode numbers, geometry parameters, initial conditions, material and medium properties. Results show excellent agreement with relative studies.

1. Introduction

Damping is an important step in understanding the dynamic behavior of various structural elements, which should be considered by designers. Inherent damping and its time-dependent variation could be presented in the form of viscoelastic material model, which should be applied on the constitutive equation. In fact, material properties such as Poisson's ratio, shear, bulk and Young's moduli are time-dependent in viscoelastic materials. However, depending on the problem circumstances, these properties could be presented in terms of time or frequency. Among the well-known models for viscoelastic materials, one may refer to the Maxwell, the Kelvin–Voight and their progressive models that have been used by many scientists, see for instance [1–6]. Power decrement series with prony series and fractional derivatives are also used in time and frequency domains, respectively [7–9]. Prony series have merits including comfortable transition between time and frequency domain and suitable curve fitting for experimental data such as [10–12] for bulk and shear moduli. A detailed review of these models for structural and vibration control can be found in [13,14].

The perfect model within the linear viscoelastic models is the Boltzmann superposition integral, which represents material behavior in arbitrary time intervals with infinitesimal steps that present any material variation. DMA results for bulk K, and shear G, moduli in conjunction with Alfrey's correspondence principle lead to the determination of Young's modulus and Poisson's ratio, as done by Kiasat et al. [10,15]. Therefore, the outputs may be embedded in the Boltzmann integral to determine time-dependent stiffness matrix components for plates or relaxation modulus and flexural rigidity for viscoelastic beam. This procedure represents the exact behavior of relaxation modulus and flexural rigidity in time or frequency domains without any simplifying assumption such as constant Poisson's ratio, bulk or shear moduli. Although these properties are deemed as constant values in various studies [16–21], this would be a disputed assumption within the viscoelastic research community, see for instance [22–27]. In an experimental theoretical research, Kiasat et al. [27] obtained the increasing behavior of the Poisson's ratio during the creep test for an epoxy compound.

Viscoelastic behavior of foundation is another striking factor for dissipating energy in continuous systems such as beam and plate on flexible foundation. Foundation viscosity is investigated in many researchers, see for instance [28–30], in which the Euler–Bernoulli beam was investigated with simply supported boundary conditions (B.Cs). Infinite Timoshenko beam on linear and nonlinear viscoelastic foundations under moving load was studied by Kargarnovin et al. [31,32]. In addition, numerical method for beam

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on viscoelastic foundation is employed to obtain solutions in time domain from the Laplace domain [33–35]. MacBain and Genin [36] and Sheng et al. [37] considered energy dissipation in viscoelastic Timoshenko beam on viscoelastic medium using complex constants for impeccable and damage states, respectively. According to literature survey, two important remarks seem to be missing. The first one is that, despite the use of higher order theories such as Timoshenko and Mindlin theories, DMA results have not been used directly for this case. Recently, Zamani and Kiasat [38] investigated free vibration of viscoelastic beams and plates on viscoelastic foundation only with simply supported B.Cs. The other B.Cs investigation is the second one that seems to be missing.

In this study, semi-analytical solutions are presented for vibration of viscoelastic Euler–Bernoulli beam and Kirchhoff–Love plate on viscoelastic medium with various B.Cs. Viscoelastic behavior of material is modeled with Boltzmann superposition integral using DMA results from Kiasat's work [10]. Viscoelastic medium behavior is modeled with parallel springs and dashpots as the Kelvin–Voight model. Effects of various parameters such as mode numbers, viscosity of medium and material are studied on the beam and plate complex natural frequencies by the iterative QZ algorithm, which is applied by Damanpack et al. [39]. For the first time, critical damping due to foundation viscosity is presented for the beam with different B.Cs. According to the complex natural frequency, unknown coefficients of dynamic response are calculated by Fourier transform. It is found that viscosity of foundation has remarkable effects on natural frequency for both beams and plates. These effects are even more critical in dynamic response of the beams, which may cause non-vibrational motion, i.e. critical or over-damped states. Finally, foundation properties and B.Cs effects are investigated on the dynamic response of beams and plates.

2. Governing equation

A rectangular thin isotropic viscoelastic plate on viscoelastic medium as shown in Fig. 1 with dimensions a, b and h as length, width, and thickness in the x, y, and z directions, respectively, is considered. Furthermore, a deformed section area of thin isotropic viscoelastic beam is depicted in Fig. 2 in which dimensions L and h_0 refer to the length and thickness in the x and z directions, respectively. A cured polymeric resin is considered for both cases of the beam and plate; also material properties are extracted from the DMA test [10].

2.1. Viscoelastic plate

In order to accurately determine the time-dependent definition of Young's modulus and Poisson's ratio of the material, the direct approach is employed. The Boltzmann superposition principle is used to define time-dependent constitutive equation for plane

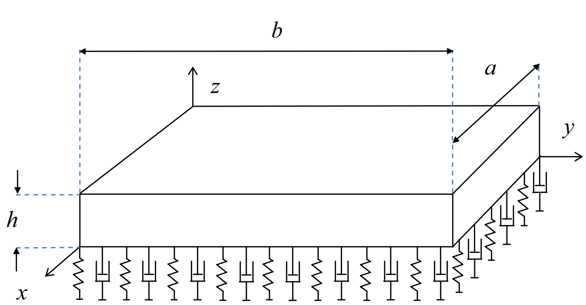

Fig. 1. Schematic and geometry of viscoelastic plate on viscoelastic medium.

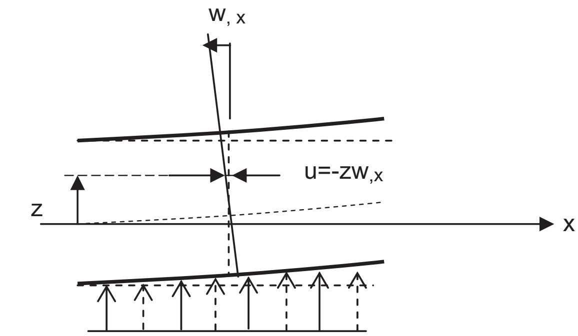

Fig. 2. Beam deformed section area on viscoelastic foundation. Solid and dashed arrows are spring and dashpot respectively.

stress state as

$$\begin{Bmatrix}\sigma_x(t)\\ \sigma_y(t)\\ \tau_{xy}(t)\end{Bmatrix}=\int_{-\infty}^{t}\begin{bmatrix}C_{11}(t-\xi) & C_{12}(t-\xi) & 0\\ C_{12}(t-\xi) & C_{11}(t-\xi) & 0\\ 0 & 0 & C_{66}(t-\xi)\end{bmatrix}\frac{d}{d\xi}\begin{Bmatrix}\varepsilon_x(\xi)\\ \varepsilon_y(\xi)\\ \gamma_{xy}(\xi)\end{Bmatrix}d\xi \tag{1}$$

where σ, ε, ξ and C_{ij} are stress, strain, Boltzmann integral variable and stiffness matrix element, respectively, which in the last case is defined for elastic material as

$$C_{ij}=\begin{bmatrix}\frac{E}{1-\upsilon^2} & \frac{\upsilon E}{1-\upsilon^2} & 0\\ \frac{\upsilon E}{1-\upsilon^2} & \frac{E}{1-\upsilon^2} & 0\\ 0 & 0 & G\end{bmatrix} \tag{2}$$

where υ and E are Poisson's ratio and Young's modulus, respectively. Applying Alfrey's correspondence principle for material properties leads to

$$s\bar{C}_{11}(s)=\frac{s\bar{E}(s)}{1-s^2\bar{\upsilon}^2(s)} \tag{3}$$

$$s\bar{C}_{12}(s)=\frac{s^2\bar{\upsilon}(s)\bar{E}(s)}{1-s^2\bar{\upsilon}^2(s)} \tag{4}$$

$$s\bar{C}_{66}(s)=s\bar{G}(s) \tag{5}$$

where s is the Laplace variable. Superior bar signifies average value of properties in Laplace domain. Simplifying and using Laplace inverse (L^{-1}) operator leads to the time dependent stiffness matrix elements as

$$C_{11}(t)=L^{-1}\left\{\frac{\bar{E}(s)}{1-s^2\upsilon^2(s)}\right\} \tag{6}$$

$$C_{12}(t)=L^{-1}\left\{\frac{s\bar{\upsilon}(s)\bar{E}(s)}{1-s^2\upsilon^2(s)}\right\} \tag{7}$$

$$C_{66}(t)=L^{-1}\left\{\bar{G}(s)\right\} \tag{8}$$

It should be noted that the Young's modulus and Poisson's ratio could not be extracted directly from DMA results. Therefore, as an alternative to DMA, the same procedure can be employed to obtain properties in the Laplace domain using

$$\bar{E}(s)=\frac{9\bar{K}(s)\bar{G}(s)}{3\bar{K}(s)+\bar{G}(s)} \tag{9}$$

$$\bar{\upsilon}(s)=\frac{1}{s}\frac{3\bar{K}(s)-2\bar{G}(s)}{6\bar{K}(s)+2\bar{G}(s)} \tag{10}$$

Substitution of Eqs. (9) and (10) in Eqs. (6) and (7) leads to time-dependent stiffness matrix elements in terms of DMA results