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Modeling cracks and inclusions near surfaces under contact loading

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ABSTRACT

In this work, a semi-analytic solution is developed for multiple cracks and inhomogeneous inclusions of arbitrary shape beneath a half-space surface subject to contact loading. The contacting surfaces can have roughness. The solution takes into account the interactions among all the inclusions and cracks as well as the interactions between them and the surface loading body. Thus, it is capable of providing an accurate description of the surface contact area and pressure and the subsurface stress field. In developing the solution, each inhomogeneous inclusion is modeled as an homogeneous inclusion with initial eigenstrain plus unknown equivalent eigenstrain using Eshelby's equivalent inclusion method; each crack of mixed modes I and II is modeled as a distribution of glide and climb dislocations with unknown densities. As a result, the inhomogeneous half-space contact problem is converted into a homogenous half-space contact problem with unknown surface contact area and pressure distribution. All the unknowns are integrated by a numerical algorithm and then determined iteratively by using the conjugate gradient method. Computational efficiency is achieved by using the fast Fourier transform algorithm. The solution is general and robust and will potentially have wide applications for reliability analysis of heterogeneous materials, in particular their wear and contact fatigue analysis.

1. Introduction

Micro-defects such as inclusions, voids and cracks in materials can not only affect their functionality and performance but also may cause their eventual failure. When these defects are located near a material surface subject to cyclic contact loading, their damage effect becomes more prominent. Such damage analysis requires an accurate knowledge of the surface contact area and pressure and the subsurface stress field in the presence of both inclusions and cracks, which however poses tremendous challenge for study.

Inclusions can be classified into homogeneous inclusions, inhomogeneities and inhomogeneous inclusions [1]. A homogeneous inclusion has the same elastic moduli as the surrounding matrix but contain eigenstrain, a generic term for inelastic strain such as plastic strain, misfit strain and thermal strain [1]. A homogeneity has different elastic moduli than the matrix but does not contain eigenstrain. An inhomogeneous inclusion not only has material dissimilarity as compared to the matrix but also contains eigenstrain. For convenience, a void can be regarded as an inhomogeneity of zero elastic moduli.

Numerous works have been reported on the study of inclusions and their interactions with other defects such as cracks, dislocations and disclinations (see e.g., [2–15]). More related works can be found in recent review by Zhou et al. [16]. However, few studies have been conducted on inclusions beneath surfaces subject to contact loading because of complexity arising from the fact that both the surface loading body and the subsurface inclusions affect the surface deformation and thus the surface pressure distribution and subsurface stress field.

The earliest work that can be found in the literature was conducted three decades ago by Miller and Keer on a two-dimensional (2D) cylindrical void or rigid inhomogeneity in a half-space under cylindrical indentation using the complex variable formulation [17]. Afterwards, almost no progress had been made in this topic for long time.

Recently, Kuo investigated multiple inhomogeneities of 2D arbitrary shape in a half-space under cylindrical indentation using the boundary element method [18]. More recently, Zhou et al. first applied Eshelby's equivalent inclusion method (EIM) [19] but bypassed the complicated Eshelby's tensor to solve multiple inhomogeneous inclusions of 3D arbitrary shape in an infinite space and in a half-space [20,21]. They then applied the same approach in conjunction with the theory of contact mechanics to solve the inhomogeneous inclusion problem with contact loading involved [22]. Leroux et al. conducted contact analysis in the presence of spherical inhomogeneities within a half-space using

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0020-7403/

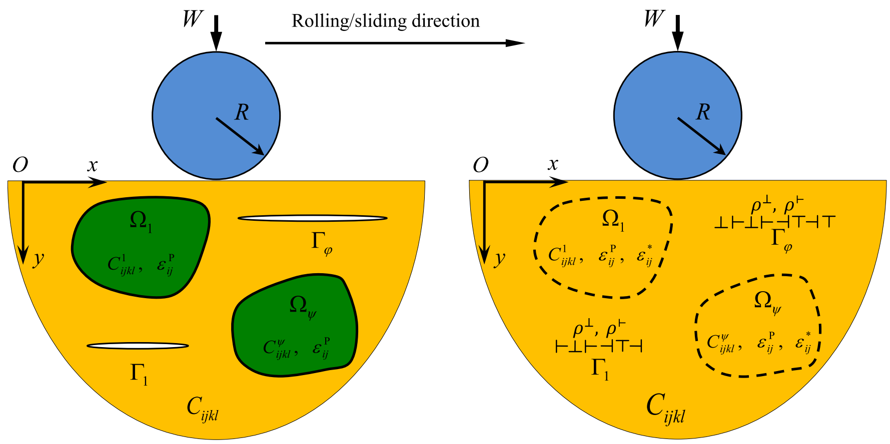

Fig. 1. (a) The original inhomogeneous half-space contact problem, and (b) the equivalent new homogenous half-space contact problem.

the EIM [23]. Chen et al. studied the interaction between dislocation and subsurface crack caused by half-plane contact [24].

On one hand, both cracks and inclusions can be formed during material manufacturing process. On the other hand, inclusions often cause stress concentration that may cause crack nucleation and propagation. Thus, the present paper aims to address such a more challenging but practical problem about multiple inhomogeneous inclusions of arbitrary shape and cracks subject to contact loading.

2. Model development

2.1. Problem description and solution approach

This study considers a 2D plane-strain problem about multiple inhomogeneous inclusions $\Omega_\psi(\psi = 1, 2, ..., n_1)$ and cracks $\Gamma_\varphi(\varphi = 1, 2, \ldots, n_2)$ near a half-space surface subject to sliding/rolling contact loading in the xOy Cartesian coordinate system, as shown in Fig. 1(a). Each inclusion Ω_ψ can be of arbitrary shape. Each crack Γ_φ is assumed to be horizontally or vertically aligned. Nevertheless, a slant crack can be modeled as a zigzag crack consisting of many small vertical and horizontal cracks. The loading body is assumed to be cylindrical and has the radius R.

The inhomogeneous inclusion Ω_ψ contains initial eigenstrain ε^P_{ij} and its elastic moduli is denoted by C^ψ_{ijkl} $(i, j, k, l = 1, 2)$; the half-space matrix has the elastic moduli C_{ijkl}. The cylindrical loading body is isotropic and has Young's modulus E_s and Poisson's ratio v_s.

When an external load W is applied on the upper loading body, a contact area is formed between it and the lower half-space. The normal pressure and tangential friction within the contact area not only causes the deformation of the two contacting surfaces but also induces the responses of subsurface cracks and inhomogeneous inclusions. These subsurface responses also change the surface deformation, which in turn changes the contact pressure and friction distributions. Thus, there exists an interaction between the surface loading body and the subsurface cracks and inhomogeneous inclusions.

The inhomogeneous half-space contact problem is solved by converting it into a homogenous half-space contact problem through the EIM and the dislocation distribution technique (DDT) [25]. Using the EIM, each inhomogeneous inclusion is modeled as a homogeneous inclusion with initial eigenstrain ε^P_{ij} plus unknown equivalent eigenstrain ε^*_{ij} to be determined; using the DDT, each crack is modeled as a distribution of climb and glide dislocations with unknown densities $\rho^\perp$ and $\rho^\vdash$ to be determined (Fig. 1(b)).

The new homogenous half-space contact problem is then decomposed into two sub-problems: (1) the half-space sub-problem to determine subsurface equivalent eigenstrains ε^*_{ij} and dislocation $\rho^\perp$ and $\rho^\vdash$ for a prescribed surface loading, and (2) the homogenous half-space contact sub-problem to determine the surface deformation, contact area and loading distribution for a given external load applied on the loading body. The two sub-problems should be correlated because of the above-discussed interaction between the surface loading body and subsurface cracks and inhomogeneous inclusions (or dislocations and eigenstrains that model them). This correlation is realized by an iterative algorithm that determines the final surface deformation due to both the surface loading body and subsurface micro-defects.

2.2. Half-space sub-problem with prescribed surface loading

When stresses within the equivalent homogeneous inclusions (Fig. 1(b)) are concerned, the utilization of Hooke's law and stress superposition gives the following governing equation:

$$C^\psi_{ijkl}C^{-1}_{klmq}(\sigma^P_{mq} + \sigma^*_{mq} + \sigma^c_{mq} + \sigma^0_{mq}) - \sigma^P_{ij} - \sigma^*_{ij} - \sigma^c_{ij} - \sigma^0_{ij} + C^\psi_{ijkl}\varepsilon^*_{kl} = 0$$
$$(\psi = 1, 2, \ldots, n_1; i, j, k, l, m, q = 1, 2) \quad \text{within} \quad \Omega_\psi \tag{1a}$$

where σ^P_{ij} is the eigenstress at a point within an inclusion caused by all the initial eigenstrains ε^P_{ij} within all the inclusions; σ^*_{ij} is the eigenstresses caused by all the equivalent eigenstrains ε^*_{ij}; σ^c_{ij} is the stress caused by all the cracks; σ^0_{ij} is the stress caused by prescribed surface loading or external loading. Eq. (1a) is similar to the governing equation obtained in the study of multiple homogeneous inclusions in half-space subjected to prescribed surface loading [21], except that it contains the extra term of σ^0_{ij}. Thus, detailed derivation is suppressed here for brevity.

In this model, the two surfaces of a crack are assumed not to be in contact with each other. When stresses along the cracks (Fig. 1(b)) are concerned, the conditions of free-surface traction should be satisfied:

$$\sigma^P_{ij} + \sigma^*_{ij} + \sigma^c_{ij} + \sigma^0_{ij} = 0, \quad (\varphi = 1, 2, \ldots, n_2; \; j = 1, 2) \quad \text{along} \quad \Gamma_\varphi \tag{1b}$$

where $i = 1$ and $i = 2$ for a crack perpendicular to the x-axis and y-axis, respectively.

Eqs. (1a) and (1b) are the governing equations for solving the inhomogeneous inclusion and crack problem. Next, numerical methods are introduced to solve them.

A computational domain is set to contain all the cracks Γ_φ and inclusions Ω_ψ and discretized into $N_x \times N_y$ square elements of the same size $2\Delta_x \times 2\Delta_y$ (Fig. 2). Each element is indexed by $[\alpha, \beta]$