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Subdivision of chatter-free regions and optimal cutting parameters based on vibration frequencies for peripheral milling process

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ABSTRACT

Considering the self-excited and forced vibrations in peripheral milling processes, a novel optimization method of cutting parameters is presented. The optimization method proposed, which is based on the vibration frequency analysis during milling processes, can achieve the most stable cutting process in relative stable region (or conditional stable region). First, relationships between vibration frequencies and phase angle of tooth of cutter in milling processes are investigated. Four kinds of spindle speeds associated with several bifurcations and vibrations are defined. Second, chatter-free regions are subdivided according to these spindle speeds. It is shown that in the so-called subregion C, cutting parameters can be simultaneously optimized for higher material removal rate (MRR) and higher surface accuracy. Third, optimal control theory is employed to determine the optimal cutting parameters, which can achieve the most stable cutting process in relative stable region. Optimizations of spindle speeds and depth of cut are conducted by using the stability charts and performance contours diagrams. The results show that the optimal cutting parameters can also be obtained in the so-called subregion C. Finally, the numerical results are verified and analyzed through milling experiments.

1. Introduction

Productivity and surface quality in milling processes have direct effects on cost, production lead time and quality of machined parts, in which vibration is one of the most common detrimental phenomena and major obstructions towards achieving automation, higher productivity, and better surface finish that is directly subjected to the dynamic characteristics of system in high-speed and high-precision milling machines. Such dynamic characteristics are dependent on the milling cutting parameters and structural parameters. Therefore, optimizations of system parameters are highly needed in order to achieve higher productivity and better surface finish.

The vibrations in milling process include three parts, i.e. free vibration, forced vibration and self-excited vibration (chatter). Chatter vibration due to the dynamic interactions between tool and workpiece has been widely studied in the past one century since Taylor first identified and described chatter in 1907. After the extensive work of Tobias [1] and Smith et al. [2], a constant time delay dynamics model for stability analysis of two-dimensional milling process using harmonic balance and infinite determines was improved [3]. A linear discrete-time model was given to analyze stability in case of low radial immersion milling [4]. Stability of milling processes with variable time delays including the effect of feed ratio was presented using the semi-discretization method [5–8]. Based on the robust stability theorem, a novel method to predict chatter-free regions for machining processes was provided, by taking in account the unknown uncertainties and changing dynamics for machining [9]. All these authors present a fundamental understanding of regenerative chatter as a feedback mechanism for the growth of self-excited vibrations due to variations in chip thickness and cutting force and subsequent tool vibration. These studies have led to graphic charts (stable chart) showing the stability information as a function of chip thickness and spindle speed; and stable region can be subdivided into so-called relative stable region (or conditional stable region) and absolute stable region (or unconditional stable region) [10].

A wide range of researches have also been conducted to determine the optimal parameters (e.g. the feed rate [11], the depth of cut [12], and the spindle speed [13]) and avoid chatter for machining, including variable helix mills [14,15], variable spindle speed [16,17] and force control [18]. Zheng et al. [19] adapted a modified FRF concept to determine the worst/best spindle speeds and the critical limiting axial depth of cut. The motivation of optimization methods established for cutting parameters in the literatures is mainly to achieve the stable cutting processes. Therefore, chatter stability is investigated in the most optimization

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methods, while the resonance due to the forced vibration has not been considered. The stability boundary has a typical 'lobed' structure. The location of stability maxima is where resonance due to forced vibration occurs [20,21]. The surface location error corresponding to the location of stability maxima is very poor [22–25]. Thus, the influences of self-excited and forced vibrations should be considered in dynamic optimization of milling system, simultaneously. Additionally, in order to achieve the most stable cutting, for optimization methods presented in literatures, the cutting parameters must be selected in the absolute stable region described in Ref. [10], where the stable limit is very low.

Thus, the present work is to establish a novel method for optimal cutting parameters in high-speed milling system considering influences of self-excited and forced vibrations. The most stable cutting can be achieved in relative stable region. The paper is organized as follows. In Section 2, dynamic models and analysis methods for milling processes, such as dynamic model, stability analysis, vibration frequencies and resonance, are introduced briefly. In Section 3, subdivision of chatter-free region is conducted based on the analysis of relationship between vibration frequencies and cutting phase difference. The optimal cutting parameters are determined in Section 4, and validation experiments are performed in Section 5. Section 6 concludes the paper.

2. Dynamic models and analysis method

2.1. Dynamic model for milling processes

Milling tool with diameter D (mm) and the number N of teeth can be considered to have two orthogonal degrees of freedom as shown in Fig. 1, a_e is the radial depth of cut, and Ω is the constant rotational angular velocity. Cutting forces excite the structure in the feed (x), and normal (y) directions, causing dynamic displacements x and y of tool in x and y directions, respectively. The governing equation for the 2-DOF oscillator has the form [22]

$$M\ddot{\mathbf{u}}(t)+C\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{F}(t)=\mathbf{S}(t)+\mathbf{Q}(t) \tag{1}$$

where, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the modal mass, damping, and stiffness matrices, respectively. The tool is assumed to be modeled as a symmetric beam; its modal parameter matrices are diagonal with the same diagonal values. $\mathbf{u}(t)=[x(t)\ y(t)]$ expresses the vibration displacements vector. $\mathbf{F}(t)$ is the cutting force vector, which decomposed in $\mathbf{S}(t)$ and $\mathbf{Q}(t)$. $\mathbf{S}(t)$ is the T periodic self-excited force vector which induces chatter vibration, and $\mathbf{S}(t)=\mathbf{S}(t+T)$. $\mathbf{Q}(t)$ is the T periodic stationary cutting force vector which induces forced vibration, and $\mathbf{Q}(t)=\mathbf{Q}(t+T)$. If there is no runout, then the periodic T of $\mathbf{S}(t)$ and $\mathbf{Q}(t)$ is the tooth passing period τ, $\tau=2\pi/(N\Omega)$; if including the runout of tool, the periodic T is spindle rotation period, $T=N\tau$.

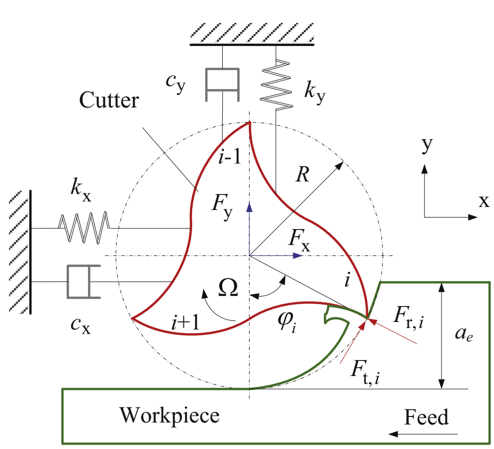

Fig. 1. 2-DOF peripheral milling model.

The self-excited force vector $\mathbf{S}(t)$ can be expressed as

$$\mathbf{S}(t)=\begin{Bmatrix} S_x(t) \\ S_y(t) \end{Bmatrix}=a_p\mathbf{k}(t)\mathbf{u}(t,t-\tau) \tag{2}$$

where, a_p is the axial depth of cut. $\mathbf{u}(t,t-\tau)=\mathbf{u}(t-\tau)-\mathbf{u}(t)$ expresses the relative displacement for successive two teeth of cutter. The elements of the so-called specific cutting force variation matrix $\mathbf{k}(t)$, which is 2×2 matrix, are

$$k_{xx}(t)=\sum_{i=1}^{N}\delta(\varphi_i(t))[K_t\cos\varphi_i(t)+K_r\sin\varphi_i(t)]\sin\varphi_i(t)$$
$$k_{xy}(t)=\sum_{i=1}^{N}\delta(\varphi_i(t))[K_t\cos\varphi_i(t)+K_r\sin\varphi_i(t)]\cos\varphi_i(t)$$
$$k_{yx}(t)=\sum_{i=1}^{N}\delta(\varphi_i(t))[-K_t\sin\varphi_i(t)+K_r\cos\varphi_i(t)]\sin\varphi_i(t)$$
$$k_{yy}(t)=\sum_{i=1}^{N}\delta(\varphi_i(t))[-K_t\sin\varphi_i(t)+K_r\cos\varphi_i(t)]\cos\varphi_i(t) \tag{3}$$

here, $\delta(\varphi_i(t))$ is a Heaviside step function that assumes a value one when the cutting tooth is engaged in cutting process and zero when the tool is out of the cut. K_t and K_r are the tangential and radial cutting coefficients, respectively. $\varphi_i(t)=\Omega t+(i-1)2\pi/N$ is the location of the ith tooth.

The stationary cutting force vector $\mathbf{Q}(t)$ is written as

$$\mathbf{Q}(t)=\begin{Bmatrix} Q_x(t) \\ Q_y(t) \end{Bmatrix}=a_p f_z\begin{Bmatrix} k_{xx}(t) \\ k_{yx}(t) \end{Bmatrix} \tag{4}$$

where f_z is the feed per tooth.

The vibration of cutter is composed of self-excited vibration (chatter) and forced vibration, and the motion is decomposed in the form

$$\mathbf{u}(t)=\mathbf{u}_p(t)+\mathbf{u}_e(t)=\begin{Bmatrix} x_p(t) \\ y_p(t) \end{Bmatrix}+\begin{Bmatrix} x_e(t) \\ y_e(t) \end{Bmatrix} \tag{5}$$

where $\mathbf{u}_p(t)=\mathbf{u}_p(t+T)$ is the forced periodic chatter free motion of the tool, and $\mathbf{u}_e(t)$ is a perturbation corresponding to the self-excited vibrations of the tool. Substitution of Eq. (5) into Eq. (1) results in the equation of forced vibration

$$\mathbf{M}\ddot{\mathbf{u}}_p(t)+\mathbf{C}\dot{\mathbf{u}}_p(t)+\mathbf{K}\mathbf{u}_p(t)=\mathbf{Q}(t) \tag{6}$$

and, the equation of self-excited vibration

$$\mathbf{M}\ddot{\mathbf{u}}_e(t)+\mathbf{C}\dot{\mathbf{u}}_e(t)+\mathbf{K}\mathbf{u}_e(t)=\mathbf{S}(t)=a_p\mathbf{k}(t)\mathbf{u}_e(t,t-\tau) \tag{7}$$

A wide range of researches have been conducted to the dynamic optimization of milling system based on chatter vibrations [12–18], which is presented in Eq. (7), delay differential equation (DDE). In the paper, the self-excited (Eq. (7)) and forced (Eq. (6)) vibrations are considered in parameters optimization of milling system, simultaneously.

2.2. Stability analysis of self-excited vibration

Stability of the machining process can be estimated theoretically via the analysis of the governing Eq. (7). Delay differential equations are usually associated with infinite dimensional phase spaces that can present several analysis difficulties. The stability properties are described by the eigenvalues of the (infinite dimensional) monodromy operator. The eigenvalues of this monodromy operator are the characteristic multipliers. The system is asymptotically stable if all the characteristic multipliers are in modulus less than 1. Since delayed systems usually have infinitely many characteristic multipliers, their stability conditions cannot be given analytically, but there exist several numerical and