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Structural analogies on systems of deformable bodies coupled with non-linear layers

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ABSTRACT

The paper is addressed at phenomenological mapping and mathematical analogies of oscillatory regimes in systems of coupled deformable bodies. Systems consist of coupled deformable bodies like plates, beams, belts or membranes that are connected through visco-elastic non-linear layer, modeled by continuously distributed elements of Kelvin–Voigt type with nonlinearity of third order. Using the mathematical analogies the similarities of structural models in systems of plates, beams, belts or membranes are obvious. The structural models consist by a set of two coupled non-homogenous partial non-linear differential equations. The problems to solve are divided into space and time domains by the classical Bernoulli–Fourier method. In the time domains the systems of coupled ordinary non-linear differential equations are completely analog for different systems of deformable bodies and are solved by using the Krilov–Bogolyubov–Mitropolskiy asymptotic method. This paper presents the beauty of mathematical analytical calculus which could be the same even for physically different systems.

The mathematical numerical calculus is a powerful and useful tool for making the final conclusions between many input and output values. The conclusions about nonlinear phenomena in multi-body systems dynamics have been revealed from the particular example of double plate's system stationary and non-stationary oscillatory regimes.

1. Introduction

Theory of Mihailo Petrović Alas, presented in two books [1,2], contains elements of mathematical phenomenology and phenomenological mapping. Both publications were published in Serbian and only a small number of his contemporaries were able to read and understand this theory. The idea of mathematical phenomenology of M. Petrović, was presented in his works entitled "Phenomenological Mapping" [1], containing the following chapters: The mapping of facts; General notation of mapping; Conventional mapping; Natural mapping; Mutual particularities of facts and Elements and properties (essentials) of facts. Alass' theory defines two tips of analogy: qualitative and mathematical analogy. "Phenomenological Mapping" by M. Petrović Alas and his mathematical phenomenology and mathematical analogy can be considered to be the continuation of the ideas of Poincare's mapping and as one of the researchers in the row leading to modern researchers who contributed to different kinds of mapping in the research of non-linear dynamics and dynamical systems. For example, nowadays in the research of non-linear dynamics, Smale's horseshoe mapping is used in the vicinity of the homoclinic unstable point.

In the time of computer and software tool expansion, Roger Penrose [3] and James Glaick [4] had similar ideas that were later applied in graphical – computer techniques. On the basis of this theory it is possible to make integration of contemporary knowledge obtained in various areas of sciences and identify analogous dynamics and phenomena. Phenomenological mapping of phenomenon and models enables multiple system dynamics models of disparate nature to be described by a single mathematical model. The well-known example is mathematical analogy of electrical circuit consisting of a resistor, an inductor, and a capacitor, connected in series or in parallel with simple harmonic oscillator. Rašković [5] gave a series of examples for electromechanical mathematically analogous vibration systems mathematically described and solved for free vibrations. By using the idea from Petrović [1,2], in the paper [7] an analogy between vector models of stress state, strain state and model of the state of the body mass inertia moments was presented and used for explanation in possible phenomenological mapping of these different kind states.

Ideas of phenomenology and phenomenological mapping, from listed references, are used for investigating dynamics, and vibration phenomena of resonance and dynamical absorption for solving

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series of research problems of dynamics of various kinds of chain and multi-body systems.

According to our knowledge, there are only several studies available in the literature discussing phenomenological mapping: in mathematics [6–9] from the area of neuroscience [10] or as a part of string theory and its applications [11]. String theory is a selfcontained mathematical model that describes all fundamental forces and forms of matter.

Composing the proper mathematical model of mechanical system presents one of the most important steps in the description of the system dynamics and structural model formulation. Where by "the description of the system" we suppose all the levels of exploring the kinetics characteristics of the systems and abilities of their improvement, control, regulation or some other usage of mechanical systems. On the other way, mathematical modeling regards on the usage of mathematical language for presentation of the behavior of practical systems. It plays the role of better understanding of systems features. Non-linearity appears both as an object's natural characteristic and the corresponding non-linearity of the systems of differential equations describing the system dynamics, which is a consequence of the choice of the coordinates of the system's description. The problem is to explore and in some possible way control non-linearity.

Theory is useful for presenting the general conclusions to the simple models while the computers are useful for obtaining the special conclusions for more complicated systems. However, in order to form a mathematical description of the complex system we are forced to introduce a number of assumptions, simplifications, neglect or possibly measure errors, and our structural model can have significant different dynamics from real physical models. To avoid this, it is useful to note the similarities in the physical phenomena and mathematical descriptions of the various systems and to take advantage in the general conclusions.

In this paper we will present mathematical models of several complex mechanical systems, introduce its analogies and explain nonlinear phenomena of passing through resonant regions. Systems consist of coupled deformable bodies like plates, beams, or membranes that are connected through discrete continuum layer with nonlinear elastic and translator and rotator inertia properties. Visco-elastic non-linear layer, with properties of translation and rotation of added mass elements, was rheologically modeled by continuously distributed elements of Kelvin–Voigt type with nonlinearity of third order with addition of rotatory elements.

The interest in the study of multi-bodies systems, as new qualitative systems, dynamics has grown exponentially over the last few years because of the theoretical challenges involved in the investigation of such systems. Recent technological innovations have caused a considerable interest in the study of component and hybrid dynamical processes of coupled rigid and deformable bodies (plates, beams and belts) (see Refs. [12–15]) denoted as hybrid systems, characterized by the interaction between subsystem dynamics, governed by coupled partial differential equations with boundary and initial conditions.

The study of transversal vibrations of multi-bodies system with elastic, visco-elastic or creep connections is important for both theoretical and pragmatic reasons. The dynamic behavior of many important structures may be investigated from mathematical models of such deformable bodies systems. The models presented in this paper may be useful in presentation of non-linear dynamics behavior for a number of real structures, for example, in civil engineering for roofs, floors, walls, in thermo and acoustics isolation systems of walls and floors constructions, orthotropic bridge decks or for building any structural application in which the traditional method of construction uses stiffened steel.

The sandwich constructions consist of two or more facing layers that are structurally bonded to a core made of material with small specific weight. This type of construction provides a structural system that acts as a crack arrest layer and that can join two dissimilar metals without welding or without setting up a galvanic cell and provides equivalent in plane and transverse stiffness and strength, reduces fatigue problems, minimizes stress concentrations, improves thermal and acoustical insulation, and provides vibration control. It is shown here that as a model of those structures it is possible to use a visco-elastically connected double deformable bodies system with non-linearity in elastic layer.

2. Partial differential equations of transversal vibrations of a double deformable bodies system

For standard rolling visco nonlinear elastic element, Fig. 1 (c) and (d) lighted on a way of the rheological models [16], we write the expressions for the velocity of translation for the center

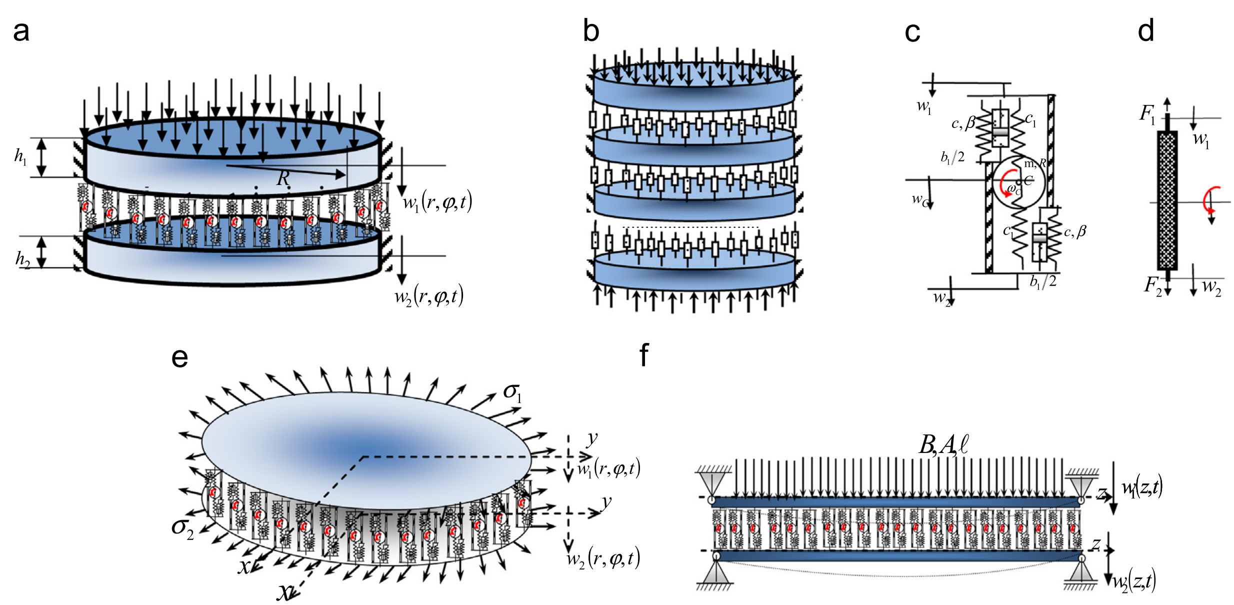

Fig. 1. (a) Double circular plate system; (b) multi-plate system; (c) the rheological model of rolling visco-elastic nonlinear discrete element; (d) the rheological scheme of rolling visco-elastic nonlinear discrete element; (f) double membrane system; and (g) double beam system.