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Elements of mathematical phenomenology of self-organization nonlinear dynamical systems: Synergetics and fractional calculus approach

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ABSTRACT

The modern dynamical systems of various physical natures, such as natural, social, economic, and technical ones, are complexes of various subsystems. They are connected by processes of intensive dynamic interaction and exchange of energy, matter, and information and incorporate nonlinear dynamics, memory, complicated transients, bifurcation and chaotic motion modes. Particularly, synergetics as a very young discipline deals with complex systems, i.e. it is concerned with the spontaneous formation of macroscopic spatial, temporal, or functional structures of systems via self-organization and is associated with systems composed of many subsystems, which may be of quite different natures. Synergetics take into account deterministic processes as treated in the dynamic systems theory including bifurcation theory, catastrophy theory, as well as basic notions of the chaos theory and develops its own approaches. Here, the fundamental basis of nonlinear theory of system's synthesis based on synergetics as well as fractional calculus approach in modern control theory together with its application will be presented. The difference of synergetic approach from the classical scientific methods is in identification of the fundamental role of *self-organization in nonlinear dynamic systems* and it is necessary to keep the conceptual correspondence to the main qualities of self-organization: nonlinearity—open systems—coherence. Synergetic approach is based on the natural homeostatic-conservation of the internal qualities of the dynamic systems of various natures. Namely, Russian scientist A.A. Kolesnikov developed a novel synergetic approach based on the ideas of modern mathematics, cybernetics, and synergetics to the synthesis of control systems for nonlinear, multidimensional and multilinked dynamic systems of various natures. The synergetic approach to control theory (synergetic control theory-SCT) is a novel nonlinear control method where the nonlinearities of a system are considered in the control design and a systematic design procedures. The invariants (synergies) and attractors, introduced as the main element of SCT, allow establishing direct link to the energy conservation laws, i.e. to the fundamental qualities of various objects. So, invariants, self-organization, and cascade synthesis are the fundamental notions of the SCT determining its essence, novelty, and content. Also, fractional calculus (FC) has a long history of three hundred years, over which a firm theoretical foundation has been established. All fractional operators consider the entire history of the process being considered, thus being able to model the non-local and distributed effects often encountered in natural and technical phenomena and they provide an excellent instrument for description of the memory, heredity, non-locality, self-similarity, and stochasticity of various materials and processes. Fractional dynamics can be encountered in various nonlinear dynamical systems such as visco-elastic materials, electrochemical processes, thermal systems, transmission and acoustics, chaos and fractals, biomechanical systems, and many others. The fractional dynamic systems with nonlinear control represent a relatively new class of applications of the FC which certified the FC as being a fundamental tool in describing the dynamics of complex systems as well as in advanced nonlinear control theory.

1. Introduction

The last decade has witnessed a tremendous attention devoted to both modeling and simulation of nonlinear dynamical systems [1–7]. This high interest is due to the growing awareness that nonlinear dynamics are inherent in a vast class of systems, phenomena, and events: natural biological systems, physical systems, engineering systems, etc. A study of the relevant state-of-the-art reveals various scientific contributions based on modeling, simulation, and control of nonlinear dynamical systems [1–4]. As we know, many things in nature don't act in linear way, i.e. whenever parts of a system

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interfere, or cooperate, or compete, there are nonlinear interactions going on and the principle of superposition fails spectacularly. Most of everyday life is nonlinear, and for nonlinear systems, there's typically no hope of finding the trajectories analytically. Even when explicit formulas are available, they are often too complicated to provide much insight. The nonlinear systems are connected by processes of intensive dynamic interaction and exchange of energy, matter, and information and incorporate nonlinear dynamics, memory, complicated transients, bifurcation and chaotic motion modes. Various interesting and striking states of nonlinear systems are of particular interest: periodic, quasi-periodic, stable, unstable, deterministic, stochastic, synchronized, torus and chaotic dynamics, etc. [5–7]. Thus, nonlinearity generates various phenomena that are difficult for us to comprehend. Nonlinearity is indispensable to create a complex system. For example, papers [8,9] provide a review of the results in the field of nonlinear mechanics which include a huge class of hybrid dynamical systems. Special attention is devoted to the development of Lyapunov's methods and the averaging theory which allowed solutions to a wide range of problems of the nonlinear dynamical systems.

On the other side, the most important feature of the nonlinear world is that disparate space–time scales (e.g., macroscopic and microscopic scales) can interfere with each other. Consequently, events of the world directly observable on our own space–time scale are, generally speaking, not closed within themselves. That is, to understand a phenomenon occurring within our human space–time scale, we must often take into account the things happening at space–time scales disparate from ours. What we investigate is the joint action of many subsystems (mostly of the same or of few different kinds) so as to produce structure and functioning on a macroscopic scale. The so-called chaos clearly exhibits consequences of this intrusion of the unknowable (at small scales) into the world we experience directly. These intrusions of the unobservable into our directly observable world make the world we wish to comprehend not self-contained. Hereby, it has become more and more evident that there exist numerous examples of nonlinear dynamical systems where well organized spatial, temporal, or spatio-temporal structures arise out of chaotic states [1–4]. The spontaneous formation of well organized structures out of germs or even out of chaos is one of the most fascinating phenomena and most challenging problems scientists are confronted with. In contrast to man-made machines, which are devised to exhibit special structures and functionings, these structures develop spontaneously - they are self-organizing. The spontaneous formation of structures, or the phenomenon of self-organization, appears in a huge variety of systems: from crystals and living cells to spiral galaxies [10,11]. Probably one of the first to use term "self-organizing" system was Ashby in 1947 [12]. The ultimate goal of genuine complex systems studies must be, from this point of view, to accomplish conceptual analysis of complexity and to construct *a phenomenology of nonlinear systems or more general-complex systems*.

In philosophy the term *'phenomenology'* is said to have appeared first in *Novum Organum*(1764) by a Swiss mathematician, Lambert (1728–1777), in the sense now being used [1]. The modern usage of word '*phenomenology*' is initiated by Husserl [13]. His motto was "*to phenomenon itself and this word was based on quite anti-metaphysical viewpoint*" (the attitude to study Nature based only on the comparison and descriptions of observable phenomena) [1,14,15]. At approximately the same time, i.e in 1911, Serbian scientist Mihailo Petrovic Alas has published his first work entitled *Elements of mathematical phenomenology* [16] and later [17] where he discussed the phenomenology of various natural, technological, and social phenomena. In book [16] he stated:

"*It happens that disparate phenomena, grouped into qualitative groups, identified by the common qualitative details of, emphasize, in the core of mutual analogy, what factors are of particular phenomenological interest that can still be used for the common phenomenological disparate phenomena or which under some circumstances, by unifying and making schematic mechanism of phenomena, thus giving them a type of mathematical analysis, thereby making possible their introduction into the problem*".

The term "*mathematical phenomenology"* began to be used at the end of the XIX century, in parallel with the development of the positivist school of philosophy by famous scientists, the Austrian physicist Ludwig Boltzmann [18] and German physicists Gustav Kirchhoff and Heinrich Hertz. Boltzmann points out at Kirchhoff and his school as a representative of the mathematical phenomenology [18]. *Mathematical phenomenology* is the presentation of the phenomenon of mathematical analogies according to Boltzmann. Since the end of the first decade of the XX century to the present, with the exception of work related to Mihailo Petrovic Alas, there are very few references in the literature on the term "*mathematical phenomenology*" [19]. However, it should be noted that there are scientists, journals, and scientific conferences that are now at the relationship of mathematics and phenomenology [15].

As can be seen from its history, in contrast to atomism, phenomenology implies that it is confined to superficial description of phenomena, never trying to go beyond them [7]. We can say 'we understand something,' or 'we know something,' when we know common features of a set described by this 'something,' if this 'something' is a common noun. When we understand something, some sort of generalization (or abstraction) is always involved and this presents the core of phenomenological understanding. Moreover, we can say the phenomenological understanding of the world may be a viewpoint that emphasizes 'essence' more than 'existence. To understand the world phenomenologically is to dissect the world into a set of phenomena each of which can be understood by a particular phenomenological framework. We could even say that a 'phenomenon realized in a certain system' is a representation of (often mathematical and abstract) 'universal structure' in terms of the concrete system. Phenomenological understanding is often detached from materials, so it has to be mathematical. In fact, to pursue phenomenological understanding is to explore a minimal mathematical structure behind a set of phenomena. If we have a reasonable mathematically expressed model of these phenomena, it is likely that a renormalization approach can extract the phenomenological theory common or universal to them [7,15]. We could even say that a 'phenomenon realized in a certain system' is a representation of (often mathematical and abstract) 'universal structure' in terms of the concrete system. Here, our intention is to present mathematical phenomenology of self-organization of nonlinear dynamical systems of various natures based on synergetics as well as fractional calculus approach, particularly in modern control theory. Many different disciplines cooperate here to find general principles governing self-organizing systems. The difference of synergetic approach from the classical scientific methods is in identification of the fundamental role of self-organization in nonlinear dynamic systems.

Synergetics is a very young discipline and many surprising results are still ahead of us. First of all, synergetics was established by Haken [20] but there are many fundamental researches to support its theoretical structure. Besides, Haken's school [20–22] of synergetics, the processes of self-organization are studied also the by Russian scientists (topics of nonlinear (autowave) processes) [23–25] as well as Brussels school of Ilya Prigorine [26,27]. Each one of the three schools arrives at the idea of self-organization from different starting points, stresses different aspects of it and, correspondingly, builds a different theoretical instrumentation. Synergetics in a narrow sense, i.e the one developed by Haken's school, puts the emphasis on the coherent behavior of the parts of the systems. According to Brussels school [26,27], self-organization is a