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Fast non-resonance rotations of spacecraft in restricted three body problem with magnetic torques

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ABSTRACT

Fast non-resonance rotations of spacecraft around center of mass in restricted three body problem with magnetic torques are considered. It is supposed that one of primary bodies has a magnetic field, the spacecraft has magnets such that the magnetic torque is constant in the frame connected with spacecraft. In addition, it is supposed that spacecraft orbit is described by quasi-periodic functions of time, the angular speed of spacecraft rotations much more than the elliptical mean motion of primary bodies.

The averaged Hamiltonian of problem is obtained. For different parameters D_j which are functionals on a set of spacecraft orbits, the evolution of angular momentum vector of spacecraft is investigated. It is shown that the increase of the magnetic torque leads to the magnification of the inclination for angular momentum vector. In limiting case when gravitational torques can be neglected, the angular momentum vector will be parallel to the plane of the rotation of primary bodies.

It is shown that there is a mathematical analogy between the purely magnetic rotations and gravitational rotations in plane orbits whenever bifurcation parameter N is unit.

1. Problem statement: Hamilton function

The investigation of spacecraft attitude motion in the central gravitational field with magnetic torques was given in [1–4]. Let us consider the non-resonance rotations of rigid spacecraft in the gravitational field of two main bodies m_1, m_2. Assume that main bodies have spherical distribution of mass, body m_1 has an exterior magnetic field, spacecraft has an infinitesimal mass, a triaxial ellipsoid of inertia and electric current systems with permanent magnets. The magnetization of spacecraft's cover and Foucault's currents can be neglected. In addition, suppose that the exterior magnetic field is simulated by oblique dipole, which creates the constant magnetic torque **P** in coordinate system connected with spacecraft, the axis of rotation motion of body m_1 has a constant orientation in inertial space, the dipole axis has a constant declination δ_m with rotation axis of m_1 and rotates together with m_1.

Suppose main bodies $m_1, m_2 (m_1 \geq m_2)$ move along elliptical orbit:

$$r = \frac{a(1-e^2)}{1+e\cos\nu},$$

where r is the distance between two main bodies, a, e, ν are the semi-major axis, eccentricity and true anomaly accordingly, the point O is the barycenter of m_1, m_2, and $\tilde{\omega}$ is the angular speed of rotation motion of m_1; then we introduce the following systems of coordinates (Fig. 1).

Let $OS_1S_2S_3$ be the inertial rectangular axes, taken along the line of apsides of elliptic motion of main bodies, perpendicular to this line and to the plane, containing two main bodies, at origin O.

$Oxyz$ is a rotating barycentric frame such that Ox passes through main bodies, Oy is perpendicular to Ox, Oz is parallel to OS_3. Its angular speed of the rotation is

$$\dot{\nu} = \frac{\omega_0(1+e\cos\nu)^2}{(1-e^2)^{3/2}} \tag{1}$$

Let $MS_2S_2S_3$ be König's frame connected with barycenter M of spacecraft such that MS_j is parallel to OS_j. Thus, König's frame moves translationally.

Suppose $Mz_1z_2z_3$ is the moving frame, taken along the principal axes of inertia for spacecraft (A, B, C are principal moments of inertia corresponding to axes Mz_1, Mz_2, Mz_3 accordingly, $A \geq B \geq C$), $MJ_1J_2J_3$ is the frame connected with angular momentum $\mathbf{I}_2$ such that MJ_3 is directed along $\mathbf{I}_2$, MJ_2 is directed along the line of the intersection of plane MS_2S_3 with the plane, which is perpendicular to $\mathbf{I}_2$, MJ_3 supplements system to right.

Let $My_1y_2y_3$ be additional König's frame such that My_3 is parallel to the rotation axis of body m_1 and has a constant angular i with MS_3 (Fig. 2), $Mx_1x_2x_3$ be the frame connected with body m_1 and rotating with angular speed $\tilde{\omega}$ such that Mx_3 is directed along the axis of magnetic dipole.

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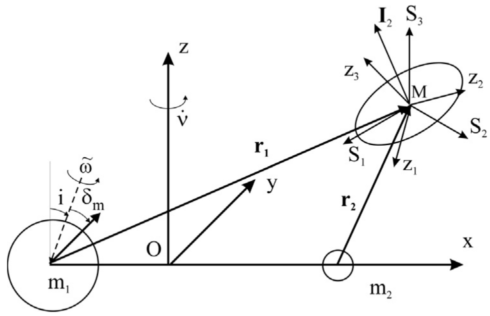

Fig. 1. Main coordinate systems.

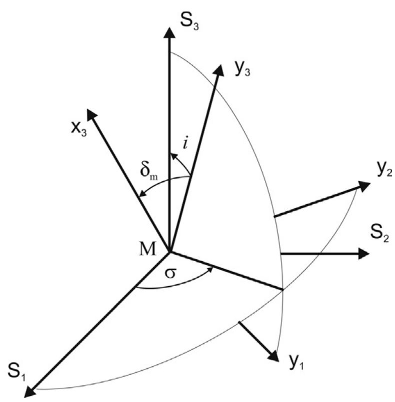

Fig. 2. Additional coordinate systems.

Let us determine the attitude of frame $Oxyz$ in inertial space $OS_1S_2S_3$ by the angle ν (true anomaly). Suppose σ is the angle of rotation around MS_3, i is the angle of rotation around My_2 (Fig. 2), λ_m is the angle of rotation of $Mx_1x_2x_3$ frame around My_3 such that $0 \le \sigma < 2\pi, 0 \le i \le \pi, \lambda_m = \tilde{\omega} t + \lambda_{0\,m}$; then we can transform $MS_1S_2S_3$ to $My_1y_2y_3$ by means of two sequential rotations on σ and i accordingly (Fig. 2). In the same way we can transform $My_1y_2y_3$ to $Mx_1x_2x_3$ with the help of rotations by λ_m and δ_m respectively.

Let $L, I_2, I_3, l, \varphi_2, \varphi_3$ be Depri-Andoyer canonical variables [5,6], which define the attitude of $Mz_1z_2z_3$ w.r.t. König's axes $MS_1S_2S_3$, and

$$\gamma_{ij} = \langle \bar{\mathbf{y}}_i, \bar{\mathbf{s}}_j \rangle,\ \Delta_{ij} = \langle \bar{\mathbf{x}}_i, \bar{\mathbf{y}}_j \rangle,\ \beta_{ij} = \langle \bar{\mathbf{j}}_i, \bar{\mathbf{s}}_j \rangle,\ \alpha_{ij} = \langle \bar{\mathbf{z}}_i, \bar{\mathbf{j}}_j \rangle,$$
$$\lambda_{1j} = \langle \bar{\mathbf{x}}, \bar{\mathbf{s}}_j \rangle,\ \lambda_{2j} = \langle \bar{\mathbf{y}}, \bar{\mathbf{s}}_j \rangle,\ \lambda_{3j} = \langle \bar{\mathbf{z}}, \bar{\mathbf{s}}_j \rangle, \qquad (2)$$

be the elements of transition matrices between frames. Here $\bar{\mathbf{s}}_i$, $\bar{\mathbf{y}}_i$, $\bar{\mathbf{x}}_i$, $\bar{\mathbf{j}}_i$, $\bar{\mathbf{z}}_i$, $\bar{\mathbf{x}}$, $\bar{\mathbf{y}}$, $\bar{\mathbf{z}}$ are the unit vectors of coordinate systems described above.

Note that, using the classical notation L, G, H, l, g, h for variables Depri-Andoyer, we get variables Serret-Andoyer [7].

It is clear that

$$\gamma_{11} = \cos\, i\, \cos\, \sigma,\ \gamma_{12} = \cos\, i\, \sin\, \sigma,\ \gamma_{13} = -\sin\, i,$$
$$\gamma_{21} = -\sin\, \sigma,\ \gamma_{22} = \cos\, \sigma,\ \gamma_{23} = 0,$$
$$\gamma_{31} = \sin\, i \cos\, \sigma,\ \gamma_{32} = \sin\, i \sin\, \sigma,\ \gamma_{33} = \cos\, i \qquad (3)$$

Using the change of variables

$$(i \to \delta_m,\ \sigma \to \lambda_m), \quad \left(i \to \delta_1,\ \sigma \to \varphi_3 - \frac{\pi}{2}\right), \quad (i \to 0,\ \sigma \to \nu),$$

we get similar formulas for Δ_{ij}, β_{ij}, λ_{ij}. Here δ_1 is the angle between MS_3 and vector $\mathbf{I}_2$, $\cos\delta_1 = I_3/I_2$.

Taking into account (2), we get the following formulas for direction cosines $\alpha_{i,j}$:

$$\alpha_{11} = -\cos\, l\, \sin\varphi_2 - \sin\, l\, \cos\, \delta_2\, \cos\, \varphi_2,\ \alpha_{12} = \cos\, l\, \cos\, \varphi_2$$
$$- \sin\, l\, \cos\, \delta_2\, \sin\, \varphi_2,$$
$$\alpha_{21} = \sin\, l\, \sin\, \varphi_2 - \cos\, l\, \cos\, \delta_2\, \cos\, \varphi_2,\ \alpha_{22}$$
$$= -\sin\, l\, \cos\, \varphi_2 - \cos\, l\, \cos\, \delta_2\, \sin\, \varphi_2,$$
$$\alpha_{31} = \sin\, \delta_2\, \cos\, \varphi_2,\ \alpha_{32} = \sin\, \delta_2\, \sin\, \varphi_2,\ \alpha_{33} = \cos\, \delta_2,$$
$$\alpha_{13} = \sin\, l\, \sin\, \delta_2,\ \alpha_{23} = \cos\, l\, \sin\, \delta_2$$

Here δ_2 is the angle between Mz_3 and vector $\mathbf{I}_2$, $\cos\, \delta_2 = L/I_2$.

Hamilton function of problem is

$$H' = \frac{I_2^2 - L^2}{2}\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right) + \frac{L^2}{2C} + U_g + U_m$$

Here

$$U_g = \frac{3}{2}\omega_0^2 a^3 \sum_{j=1}^{2} \frac{\mu_j}{r_j^3}[(B-A)\gamma_{j2}^{*2} + (C-A)\gamma_{j3}^{*2}]$$

is a force function of gravitational torques [1], U_m is a force function of magnetic torques acting on spacecraft:

$$\gamma_{ij}^* = \frac{1}{r_i}[(\beta_{k1}\ \cos\, \nu + \beta_{k2}\ \sin\, \nu)(x - x_i) + (\beta_{k2}\ \cos\, \nu - \beta_{k1}\ \sin\, \nu)y + \beta_{k3}z]\alpha_{jk}$$

is the direction cosine between $\mathbf{r}_j$ and Mz_i-axis, f is an universal gravitational constant, $\omega_0^2 = [f(m_1+m_2)/a^3]$ is the mean motion of main bodies, $\mu_1 = 1-\mu$, $\mu_2 = \mu$,

$$\mu = m_2/(m_1+m_2),\ r_i = \sqrt{(x - x_i^*)^2 + y^2 + z^2},\ x_1^* = -\mu_2 r,\ x_2^* = \mu_1 r$$

Here and below the summation is taken over lower indexes that repeat twice.

Let us construct the force function U_m. Suppose the magnetic dipole rotates together with body m_1, $\mathbf{H}$ is a magnetic field strength creating by dipole; then from [9], it follows that

$$\mathbf{H} = \frac{\mu_m}{r_1^3}[3\langle \bar{\mathbf{x}}_3, \bar{\mathbf{e}}_{r_1} \rangle \bar{\mathbf{e}}_{r_1} - \bar{\mathbf{x}}_3]$$

Here μ_m is the magnitude of dipole magnetic torque, $\bar{\mathbf{x}}_3$ is the unit vector directed along the dipole axis, $\bar{\mathbf{e}}_{r_1}$ is the basis vector directed along $\mathbf{r}_1$.

The force function of magnetic torques is

$$U_m = \langle \mathbf{P}, \mathbf{H} \rangle = \frac{\mu_m}{r_1^3}[3\langle \bar{\mathbf{x}}_3, \bar{\mathbf{e}}_{r_1} \rangle \langle \mathbf{P}, \bar{\mathbf{e}}_{r_1} \rangle - \langle \mathbf{P}, \bar{\mathbf{x}}_3 \rangle], \qquad (4)$$

where $\mathbf{P}$ is the constant magnetic torque which created by spacecraft's magnets.

Using (2), we get

$$\mathbf{P} = P_i \alpha_{ik} \beta_{kl} \bar{\mathbf{s}}_l, \quad \bar{\mathbf{x}}_3 = \Delta_{3j} \gamma_{jk} \bar{\mathbf{s}}_k, \quad \bar{\mathbf{e}}_{r\,1} = \xi_\kappa \bar{\mathbf{s}}_k \qquad (5)$$

Here P_i is the constant projection of vector $\mathbf{P}$ on the Mz_i-axis $(i = 1, 2, 3)$,

$$\xi_k = \frac{[(x+\mu)\lambda_{1k} + y\lambda_{2k}]}{r_1}, \quad \xi_3 = \frac{z}{r_1}, \quad k = 1, 2$$

Substituting (5) in (4), we get

$$U_m = \frac{\mu_m}{r_1^3} P_i \alpha_{ik} \beta_{kl} \Delta_{3j} [3\xi_l \gamma_{js} \xi_s - \gamma_{jl}]$$

Let us consider that U_m and U_g are comparable on magnitude.

Note that the skew dipole model does not account the quadrupole part of the magnetic field, which can be significant for the satellite close to the planet with mass m_1 [10]. The influence of the quadrupole components on the dynamics of a charged satellite is investigated in [11,12].

Suppose the attitude motion of spacecraft have nothing influence over its orbital motion; then the orbit of spacecraft is considered as a known quasi-periodic function of time in frame $Oxyz$:

$$x = \sum_{\|p\| \ge 0} C_p^{(1)} \exp\, i\langle \boldsymbol{p}, \boldsymbol{\omega} \rangle t, \quad y = \sum_{\|p\| \ge 0} C_p^{(2)} \exp\, i\langle \boldsymbol{p}, \boldsymbol{\omega} \rangle t,$$