International Journal of Non-Linear Mechanics 73 (2015) 75–84

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Generaleristics of unsteady MHD temperature boundary layer

Dragiša Nikodijević, Živojin Stamenković*

Faculty of Mechanical Engineering, University of Nis, Nis, Serbia

ARTICLE INFO

Article history:
Received 28 March 2014
Received in revised form
9 August 2014
Accepted 4 November 2014
Available online 13 November 2014

Keywords:
MHD
Temperature
Boundary layer
Similarity parameters

ABSTRACT

Unsteady temperature magnetohydrodynamic (MHD) boundary layer on the arbitrary shape body which temperature is function of time and longitudinal coordinate has been studied. A generalized similarity method is applied to solve the governing non-linear differential equations. Universal partial differential equations of the discussed problem are obtained using the method by which the equations are transformed in such a way that neither the equations nor the boundary conditions depended on particular data of the considered problem. These equations are solved numerically and the integration results are given for different values of introduced parameters. Those solutions permit, before their application to the particular cases (assigned free stream velocity), a generalized analysis of the effects of certain pertinent parameters to the development of the boundary layer. Obtained results are discussed in detail for different values of introduced parameters. Based on the given universal solutions result it is possible to calculate any particular problem for defined free stream velocity and body temperature distribution.

1. Introduction

The phenomenon of separation is one of the most interesting features of the motion of an incompressible fluid past a solid body. Separation is usually an undesired flow feature in engineering applications, which would bring a loss of lift, an increase of drag, and also diminish pressure recovery etc., therefore, a considerable amount of research efforts has been devoted to the control of flow separation. As early as the beginning of the 20th century, Prandtl [1] started to study the flow separation control, and developed his famous boundary-layer theory. Since then, wide varieties of experimental and numerical studies have been performed by Tsinober [2], Lee et al. [3], Bajerm et al. [4], and Atik et al. [5], to determine efficient and feasible control approaches for wall shear stress modification.

Usually flow control involves passive and active devices. The passive devices such as riblets, splitters and control cylinders and active mechanisms such as suction, blowing, injection of micro-bubbles and polymeric fluids, thermal strips, acoustic excitations and rotary oscillation have been the subjects of many investigations, Kwon and Choi [6], Lecordier et al. [7], Li et al. [8], Posdziech and Grundmann [9]. Active control methods have attracted more attention in recent years, e.g. optimal cylinder flow control (Sekhar et al. [10], Hui et al. [11]), suction and blowing-Pierre and Fahad [12], thermal effect, Xin et al. [13] and so on.

The flow of electrically conducting fluid in presence of magnetic field is very important from the technical point of view and such types of problems have received much attention by many researchers because of the potential to control the flow. In these cases the flow control can be realized by the Lorentz force, and this active control method has been recognized fifty years ago. For the purpose of boundary layer control on a flat plate, Gailitis and Lielausis [14] first proposed the conductive approach, where a streamwise Lorentz force was applied on the laminar boundary layer in order to increase the thrust and delay the transition to turbulence by preventing the increase of the laminar boundary layer thickness.

The hydrodynamic behavior of boundary layer along the flat plate in the presence of a constant transverse magnetic field was first analyzed by Rossow [15], who assumed that magnetic Reynolds number is so small that the induced magnetic field could be neglected. Sparrow and Cess [16] discussed the effect of a magnetic field on free convection heat transfer on isothermal vertical plate. Gupta [17] studied laminar free convection flow of an electrically conducting fluid past a vertical plate with uniform surface heat flux and variable wall temperature in the presence of a magnetic field. Riley [18] investigated the flow of an electrically conducting fluid on a vertical plate in the presence of strong magnetic field applied normal to the flow. Hossain [19] considered the MHD free convection flow with viscous and Joules heating effects past a semi infinite plate. Watanabe and Pop [20] explained the Hall effects on an MHD boundary layer flow over a continuous

* Corresponding author.
E-mail addresses: dragisan@masfak.ni.ac.rs (D. Nikodijević), zikas@masfak.ni.ac.rs (Ž. Stamenković).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.11.016
0020-7462/

Nomenclature

Roman letters

B	magnetic induction
c_p	specific heat capacity
D	standardization constant
Ec	Eckert number
F	characteristic function, $F = U\partial z/\partial t$
$f_{k,n}$	dynamical parameters
g	time derivative of function z
$g_{k,n}$	magnetic parameters
h	characteristic linear scale
H	characteristic function, $H = \delta^*/\delta^{**}$
H^*	characteristic function, $H = \delta^*/h$
H^{**}	characteristic function, $H = \delta^{**}/h$
$l_{k,n}$	temperature parameters
N	interaction parameter, $N = \sigma B^2/\rho$
Pr	Prandtl number
q	temperature difference
t	time
T	thermodynamic temperature
u	longitudinal velocity
v	transversal velocity
U	free stream velocity
x	longitudinal coordinate
y	transversal coordinate
z	characteristic function, $z = h^2/\nu$

Greek symbols

δ^*	displacement thickness
δ^{**}	momentum thickness
Φ	dimensionless stream function
η	dimensionless transversal coordinate
λ	thermal conductivity
μ	viscosity
ν	kinematic viscosity
Θ	dimensionless temperature difference
ρ	fluid density
σ	electrical conductivity
τ	shear stress
Ψ	stream function
ξ	characteristic function, $\xi = \tau_w h/(\mu U)$
ξ_t	dimensionless temperature gradient

Subscripts

0	initial time moment
1	known boundary layer cross-section
∞	outer boundary of boundary layer
w	body surface

moving flat plate. Ramesh et al. [21] studied MHD boundary layer flow over a non-linear stretching boundary with suction and injection, while Hsiao [22] gave research results of MHD mixed convection for viscoelastic fluid past a porous wedge. Recently Atmani et al. [23] study the three-dimensional separation of boundary layer over blunt bodies.

The aim of this paper is to study unsteady temperature, two-dimensional laminar MHD boundary-layer of incompressible fluid. Few analytical methods are evolved till now to calculate the separation of boundary layer over arbitrary shape bodies, even over simple configuration. Some methods more or less approached generally use empirical values, which are obtained, from experimental measures. However, the usual approximate method of the integral thickness of Karman and Polhausen is well manageable, useful and practical. However, it does not have a systematic base in order to obtain an extension by taking into account the complexity that the obstacles and body forces can sometimes present. Loitsianski [24] suggested a parametric method (generalized similarity method) with a steady mathematical base, developed after that by Bushmarin and Saraev [25], and Saljnikov [26].

The method consists of universalization equations of boundary layer in the way that neither the equations nor the boundary conditions depended on particular data of the considered problem. Thanks" to the transition from mechanical computing devices to electrical computers today is possible to carry out a numerical integration of parametric equations. The beginning of the story about "Computing in Serbia" starts with Mihailo Petrovic Alas [27] hydro integrator that he constructed, and presented at the Exposition Universelle in Paris in 1900, and received the Gold Medal for it. The hydro integrator was based on the principle of hydraulic analogy, and it could solve two classes of differential equations. In addition, it was a kind of predecessor of the plotters, because calculation data was written automatically with a pen on the paper rolled around the cylinder.

In this paper, the three sets of parameters and one constant parameter are used to transform the full governing partial differential equations into a system of equations and corresponding boundary conditions, which form is unique for all particular problems and this form is considered as universal. Obtained system of equations are then solved numerically using three-diagonal method, known in the East literature as the "progonka" method. Part of representative results obtained with numerical are presented for some characteristic values of the governing parameters.

2. Problem formulation

Consider the unsteady MHD flow of a viscous and incompressible fluid around a two-dimensional body of arbitrary shape that coincides with the plane $z = 0$. The Cartesian coordinates $x, y, z$are taken with the origin O at the body surface and are defined such that the x-axis is measured along the body, the y-axis is measured in the transversal direction. Velocity of flow is considered much lower then speed of light and usual assumption in temperature boundary layer calculation that temperature difference is small (under 50 °C) is used, accordingly physical properties of fluid are constant (viscosity, heat conduction, electrical conductivity, magnetic permeability, specific heat capacity …). Body surface temperature is a function of time and longitudinal coordinate$x$.

We assume that a uniform magnetic field of strength B is applied normal to the body surface. It is considered that the magnetic Reynolds number is small, i.e. $Rm = \mu_0 \sigma U L \ll 1$, where μ_0 is the magnetic permeability, $\sigma$is the electrical conductivity and U,L are the characteristic free stream velocity and length, respectively. Under these conditions, we can neglect the effect of the induced magnetic field in comparison to the applied magnetic field.

Introduced assumptions partly simplify considered problem, however obtained physical model that include viscous and Joule heating is interesting from practical point of view, because its