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Chaotic behavior of a body in a resistant medium

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A R T I C L E I N F O

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A B S T R A C T

We study the pitch motion dynamics of a rigid body in a resistant medium under the influence of a biharmonic torque $a\ \sin\ \theta + b\ \sin\ 2\theta$. Such nutation angle dependence of the biharmonic aerodynamic torque is typical for uncontrolled re-entry vehicles of segmentally conical, blunted conical, and other shapes (Soyuz, Mars, Apollo, Viking, Galileo Probe, Dragon). The presence of the second harmonic in the biharmonic torque is the cause of additional unstable equilibrium. In case of spatial motion a small perturbation is a small difference of the transverse inertia moments of the body. In this case, two Euler angles $\theta$and ψ are the positional coordinates, and we can observe a chaos. In case of the planar motion the body is perturbed by a small aerodynamic damping torque and a small periodic torque of time. We show by means of the Melnikov method that the system exhibits a transient chaotic behavior. This method gives us an analytical criterion for heteroclinic chaos in the planar motion and an integral criterion for the spatial motion. The results of the study can be useful for studying the chaotic behavior of a spacecraft in the atmosphere.

1. Introduction

The dynamics of rotating bodies is a classic topic of study in mechanics. In the eighteenth and nineteenth centuries, several aspects of the motion of a rotating rigid body were studied by such famous mathematicians as Euler, Cauchy, Jacobi, Poinsot, Lagrange, and Kovalevskaya. In some cases, for the study of dynamical systems it can be useful to use elements of mathematical phenomenology and phenomenological approximate mappings for obtaining approximate differential equations and approximate solutions in local area around singular points, linear and nonlinear approximations [1–2]. However, the study of the dynamics of rotating bodies is still very important for numerous applications such as the dynamics of satellite gyrostat, spacecraft, re-entry vehicle, and the like. Note that only some of the papers are devoted to the modern problem of rigid body dynamics. So in an independent way, Sadov [3] first obtained sets of action-angle variables for the rotational motion of a triaxial rigid body. Deprit and Elipe [4] used Sadov's variables to convert directly the Serret–Andoyer variables [5–7] into action-angle variables, thereby making Hamiltonian dependent on only two momenta. Akulenko et al. [8] considered perturbed motion about a fixed point of a dynamically symmetrical heavy solid in a medium with linear dissipation and obtained an averaged system of equations. Yaroshevskii created fundamentals of the dynamics of re-entry vehicles, which were used for designing the Soviet spacecraft such as Vostok, Souz, Luna, Venera and Mars. Yaroshevskii wrote two books in Russian and a large number of articles on this problem the latter of which [9–12]. Aslanov [13] studied the motion of a rotating rigid body in the atmosphere of a planet under the action of a restoring torque which depends on time and the angle of nutation. The rigid body (re-entry vehicle) intended to descend into the atmosphere usually has a small aerodynamic and dynamic asymmetry, for example, it has a small relative difference between the transverse moments of inertia [14]. In this case, the angular motion depends on two Euler angles: the nutation angle θ (spatial angle of attack) and the angle of spin ψ. If the frequency of change of these angles becomes multiple to the relation of simple integers, then a parametrical resonance occurs [14]. Holmes and Marsden applied the methods of chaotic dynamics [15] for solving a similar problem. Holmes and Marsden considered the problem of spatial motion of the heavy rigid body with a small dynamic asymmetry when the torque of gravity was proportional to $m_\theta \sim\ \sin\ \theta$. Similar tasks have also been discussed in the papers [16–19].

This paper focuses on the study of the motion of a blunt rigid body in an atmosphere which is under the action of a biharmonic aerodynamic torque $a\ \sin\ \theta + b\ \sin\ 2\theta$. The purpose of the paper is the finding of the conditions of existence of chaos in motion in the slightly asymmetrical rigid body in the atmosphere under the action of small perturbations and determining the influence of chaos on the behavior of the rigid body.

The paper is divided into five sections. In Section 2 the statement of the problem is given. In Section 3 the spatial motion of the slightly asymmetrical rigid body about its center of mass in an atmosphere is considered. An aerodynamic torque on the body

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is determined by the biharmonic dependence on the angle of nutation. Hamilton's canonical equations are derived and conditions are found for the existence of unstable equilibria of the system. Homoclinic orbits are determined in an analytical form and Melnikov function is constructed in the modification of Holmes and Marsden [15]. Numerical simulation of a chaotic behavior of the system completes the section. In Section 4 we find an exact analytical representation of the Melnikov function for the planar motion, if the small disturbance is determined as the sum of a periodic time function and a dissipative torque. The analytical results given by the Melnikov method have been confirmed by a good agreement with direct numerical calculations in the construction of Poincaré sections by using the fourth-order Runge–Kutta algorithms. In Section 5, it is concluded that the biharmonic system will exhibit a lot of chaotic motions due to the combined physical parameters with external torques that are dissipative and periodic or due to the small dynamic asymmetry.

2. Problem formulation

Let's determine a place of the considered problem in the general problem of rigid body dynamics and also note an analogy to the motion of a heavy rigid body and the rigid body in the resisting medium (atmosphere of a planet). Gravity and aerodynamic torques acting on the sphere with a displaced center of mass in the resisting medium are proportional to $\sin\,\theta$ (Fig. 1a and b). The shape of the Soviet spacecraft Vostok was a sphere. On board Vostok, Soviet cosmonaut Yuri Gagarin made history on April 12, 1961, when he became both the first person in the world to enter space and to return to Earth. However, the modern re-entry vehicles have a blunted conical shape (Apollo, Galileo Probe, Dragon), it is to provide efficient braking in the atmosphere. For these re-entry vehicles (Fig. 1c) the aerodynamic torque is well approximated by biharmonic dependence on the nutation angle

$$m_\theta = a' \sin\,\theta + b' \sin\,2\theta \tag{1}$$

However, the dependences on the angle of nutation (1) can have three positions of equilibrium, and one of them is unstable. The stable position at the points $\theta_* = 0$ and $\theta_* = \pi$; and unstable in the third intermediate point $\theta_* \in (0, \pi)$ [13,20,21]. The presence of the second harmonic in (1) causes the possibility of appearance of an additional equilibrium position – saddle point on a phase portrait. For the considered spacecraft position $\theta = 0$ is stable; therefore, a derivative of the function $m_\theta\,(\theta)$ with respect to the angle θ at this point is negative

$$\left.\frac{dm_\theta}{d\theta}\right|_{\theta=0} = (a' \cos\,\theta + 2b' \cos\,2\theta)|_{\theta=0} < 0 \tag{2}$$

or

$$2b' < -a' \tag{3}$$

And if there exists an intermediate position of equilibrium inside the interval of $(0, \pi)$, then

$$m_\theta(\theta) = \sin\,\theta\left(a' + 2b' \cos\,\theta\right) = 0 \tag{4}$$

which holds true, if

$$|2b'| > |a'| \tag{5}$$

It is obvious that (3) and (4) are valid simultaneously when $b' < 0$. Note that the dependence of $m_\theta(\theta)$ given in Fig. 1 satisfies conditions (3) and (4). The stable position occurs not only in the point of $\theta = 0$, but also in the point of $\theta = \pi$ when (3) is fulfilled for the re-entry vehicle. The motion of the spacecraft in a neighborhood of $\theta = \pi$ cannot be allowed, because in this case the back part of the body will move towards an approach flow. A simultaneous existence of the unstable equilibrium positions and small perturbations can lead to chaos.

The role of small perturbations may play, for instance, a small dynamic asymmetry of the body or a small external torque. The rigid body with a triaxial ellipsoid of inertia possesses a small dynamic asymmetry, if its transverse inertia moments differ little from each other. Then the small dynamic asymmetry is written as

$$\varepsilon = (I_2 - I_1)/I_1 \tag{6}$$

where ε is a small parameter.

Small disturbance torque is represented as the sum of the periodic term and dissipative term

$$M_d = \left(\nu\,\cos\,\omega t - \delta\dot{\theta}\right)I_1 \tag{7}$$

where ν and $\delta > 0$ are small parameters, $\omega$and t are frequency and time, respectively.

Below we consider successively two separate problems of perturbed motion: the problem of a spatial motion of the body with a small asymmetry (6) and the problem of a planar motion of the body under the external torque (7).

3. The spatial motion of the asymmetrical body

3.1. Hamiltonian equations

Consider the spatial motion of the rigid body about its center of mass in an atmosphere. To suppose that the biharmonic torque acts on the rigid body

$$m_\theta = aI_1\,\sin\,\theta + bI_1\,\sin\,2\theta \tag{8}$$

where

$$a = a'/I_1, \quad b = b'/I_1 \tag{9}$$

Kinetic energy and potential energy of the spacecraft in this case become

$$\begin{aligned} T &= \frac{1}{2}\left(I_1 p^2 + I_2 q^2 + I_3 r^2\right) \\ &= \frac{1}{2}\Big[I_1\left(\dot{\phi}\,\sin\,\theta\,\sin\,\psi + \dot{\theta}\,\cos\,\psi\right)^2 \\ &\quad + I_2\left(\dot{\phi}\,\sin\,\theta\,\cos\,\psi - \dot{\theta}\,\sin\,\psi\right)^2 + I_3\left(\dot{\phi}\,\cos\,\theta + \dot{\psi}\right)^2\Big] \end{aligned}$$

$$\Pi = -\int M_\theta d\theta = aI_1\,\cos\,\theta + bI_1 \cos^2\theta$$

where (p, q, r) are rotation components in the body frame and (ϕ, ψ, θ) are Euler angles. Then the Hamiltonian is

$$\begin{aligned} H = T + \Pi &= \frac{\left[(p_\phi - p_\psi\,\cos\,\theta)\,\sin\,\psi + p_\theta\,\sin\,\theta\,\cos\,\psi\right]^2}{2I_1 \sin^2\theta} \\ &+ \frac{\left[(p_\phi - p_\psi\,\cos\,\theta)\cos\,\psi - p_\theta\,\sin\,\theta\,\sin\,\psi\right]^2}{2I_2 \sin^2\theta} \\ &+ \frac{p_\psi^2}{2I_3} + aI_1\,\cos\,\theta + bI_1 \cos^2\theta. \end{aligned} \tag{10}$$

where $\left(p_\phi = \partial T/\partial\phi, \quad p_\psi = \partial T/\partial\psi, \quad p_\theta = \partial T/\partial\theta\right)$ are the generalized momentums and (θ, ψ, ϕ) are the generalized coordinates.

The Hamiltonian (10) can be written as

$$H = H^0 + \varepsilon H^1 + O(\varepsilon^2) \tag{11}$$

where

$$H^0 = \frac{p_\theta^2}{2I_1} + \frac{\left(p_\phi - p_\psi\,\cos\,\theta\right)^2}{2I_1 \sin^2\theta} + \frac{p_\psi^2}{2I_3} + aI_1\,\cos\,\theta + bI_1 \cos^2\theta \tag{12}$$

$$H^1 = -\frac{\left[(p_\phi - p_\psi\,\cos\,\theta)\cos\,\psi - p_\theta\,\sin\,\theta\,\sin\,\psi\right]^2}{2I_1 \sin^2\theta} \tag{13}$$