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Wave propagation in layer with two preferred directions

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ABSTRACT

This article is concerned with overall or macroscopic properties of a composite material with no distinction made between the fibres and the matrix which they are embedded in. All the properties with dimensions larger than the fibre diameter and spacing are regarded as averaged over a volume of material. The systems of particular interest here are in the fibre reinforced composites with the fibres being very much stiffer and stronger than the matrix.

Laminated plates of fibre-reinforced material are often fabricated from prepreg tapes, laid up according to some specific arrangement of fibre orientation and then bonded together. An angle-ply laminate is formed by alternating plies so that the families in adjacent laminas are inclined by angle ϕ and $-\phi$ to given direction alternately. The process of fabricating a multilayered plate of this material gives rise to a laminate in which the plies are separated by resin rich layer, and when this layer is thin enough that its thickness is negligible it may be regarded as plate reinforced by two families of fibres. Problems shall be considered in three dimensions, but attention shall be restricted to linear elasticity theory. The plate under consideration is reinforced by two mechanically equivalent families of fibres, but with no other preferred directions, so that it is locally orthotropic with respect to the plane of the fibres and to the two planes that orthogonally bisect the fibres.

In this article linear elastic stress–strain relation is employed to derive dispersion curves for plane harmonic waves propagating in a plate of finite thickness but of infinite lateral extent. Attention is restricted to waves propagating in the plane parallel to stress free plate faces where waves travelling at any angle to one of the families of very strong fibres are examined. The dispersion equations, relating the phase velocity to the wavelength, are obtained. The fundamental modes are examined for symmetric as well as for anti-symmetric deformations. This leads to full understanding of displacement field as well as stress field.

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1. Introduction

In recent years there has been considerable activity in the study of the mechanical behaviour of composite materials and possibilities of their applications. For example, aluminium alloys are used in advanced applications because their combination of high strength, low density, durability, machinability and cost are very attractive. However, using aluminium matrix composite materials may considerably extend the scope of these properties. One may come to similar conclusion by considering polymer matrix composites.

Here we are concerned with fibre-reinforced composite materials that have an important property that they are anisotropic, and in many cases this anisotropy may be very strong, in the sense that mechanical properties are strongly dependent on direction. They usually consist of one or more filamentary phases embedded in a continuous matrix phase. Such materials are highly resistant to deformation by extension in the fibre direction compared to other deformation modes. The use of fibre reinforced composites is prevalent in modern structures, especially those for which a high strength to weight ratio is of primary concern.

Generally, composite material must be man-made as a combination of at least two chemically distinct materials with a distinct interface separating constituents. It should create properties, which could not be obtained by any of the constituents on its own.

The continuous fibre reinforced composites have as their main features improvement of stiffness and strength, reduction of wear

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and creep, anisotropic properties, improved strength in fibre direction, high price and complex manufacturing techniques.

The discontinuously reinforced composites are developed, when strength is not the main objective, but when a better wear resistance, a controlled thermal expansion, and a higher service temperature are expected.

The mechanical behaviour of unidirectional reinforced composites is relatively well understood. The tensile behaviour of the composite can be predicted from the behaviour of individual constituents by simple rules of mixture and simple structural mechanics to describe off-axis behaviour.

Fibre reinforced materials are materials with preferred directions and their macroscopic behaviour has been described as a continuum model by Spencer [1]. For materials with very strong fibres mathematic description is considerably simplified by assumption that fibres are inextensible. Detailed analysis of deformation fields for such materials is given by Spencer [2]. Dynamic behaviour of plate reinforced by one family of fibres is described by Green [3], and Green and Milosavljević [4], Bending waves of plate reinforced by two families of fibres and surface waves are described in Refs. [5,6] by Milosavljević, respectively. Dispersion relations in long wave limit in plate reinforced by two families of extensible fibres are derived in [7], and in plate reinforced by two families of inextensible fibres are derived in [8] by Milosavljević. Rheological model of circular cylindrical sandwich plate system has been considered by Hedrih and Simonovic [9], which is analogous to considering laminate structures, which will be extension of present study of one highly anisotropic layer.

In this paper we consider wave propagation in plate reinforced by two families of inextensible fibres in an arbitrary direction, parallel with the stress free boundaries. Dispersion relations for both symmetric and anti-symmetric fundamental modes are obtained. Particular attention is given to waves propagating perpendicular to one of the family of fibres.

2. Materials reinforced by strong fibres

Formulation of tangent modules in direct notation enables description independently of coordinate system and discussion of their properties in spatial as well as in material descriptions.

General definition of tangent modulus is based on existence of strain energy function of an elastic solid. To a large extent notation used by Spencer [1] is followed. All quantities are referred to a fixed Cartesian coordinate system, and all vector and tensor components are components in this system.

Suppose that a body of an elastic material undergoes a deformation in which a typical particle which in reference configuration initially has position vector $\mathbf{X}$, with components X_R, $(R=1,2,3)$, at subsequent time t moves to the point with position vector $\mathbf{x}$ with components x_i, $(i=1,2,3)$. Then the deformation is described by equations of the following form:

$$\mathbf{x}=\mathbf{x}(\mathbf{X},t) \quad \textit{or} \quad x_i=x_i(X_R,t), \tag{2.1}$$

and for fixed time t, the three functions define deformation from the configuration $\mathbf{X}$, to $\mathbf{x}$. The particle displacement $\mathbf{u}$ from the reference configuration to the present one is defined by the following equation:

$$\mathbf{u}=\mathbf{x}-\mathbf{X}, \quad \textit{or} \quad u_k=x_k-\delta_{kR}X_R. \tag{2.2}$$

The local properties of the deformation are characterised by the nine deformation gradients of tensor $\mathbf{F}$ which has components $F_{iR}=\partial x_i/\partial X_R$, and right Cauchy–Green tensor $\mathbf{C}$ has the following form:

$$\mathbf{C}=\mathbf{F}^{\mathbf{T}}\cdot\mathbf{F}=\mathbf{I}+2E, \quad C_{RS}=F_{iR}F_{iS}=\frac{\partial x_i}{\partial X_R}\cdot\frac{\partial x_i}{\partial X_S}=\delta_{RS}+2E_{RS}, \tag{2.3}$$

in which E_{RS} represents components of Green Lagrange strain tensor $\mathbf{E}$, given as follows:

$$E_{RS}=\frac{1}{2}\left(\delta_{iR}\frac{\partial u_i}{\partial X_S}+\delta_{iS}\frac{\partial u_i}{\partial X_R}+\frac{\partial u_i}{\partial X_R}\frac{\partial u_i}{\partial X_S}\right). \tag{2.4}$$

Left Cauchy–Green strain tensor $\mathbf{B}$ (Finger tensor), is defined as follows:

$$\mathbf{B}=\mathbf{F}\cdot\mathbf{F}^{\mathbf{T}}, \quad B_{ij}=F_{iR}F_{jR}=\frac{\partial x_i}{\partial X_R}\cdot\frac{\partial x_j}{\partial X_R}. \tag{2.5}$$

If δV and δv are the volumes of material volume element in reference and deformed configuration, respectively, ρ_0 and ρ densities of the element in these configuration, then it follows $\delta v/\delta V=\rho_0/\rho=\det\mathbf{F}$.

The strain energy function (per unit volume of the un-deformed body), in the case of isotropy, has following form:

$$\psi=\psi(\mathbf{C}), \tag{2.6}$$

and, therefore, tangent modulus may be constructed to satisfy the corresponding constitutive rate equations. Accordingly, if there exists the strain energy function, the second Piola–Kirchhoff's stress tensor $\mathbf{S}$ and elastic stiffness tensor in material description may be written, respectively,

$$\mathbf{S}=2\frac{\partial\psi}{\partial\mathbf{C}}=\frac{\partial\psi}{\partial\mathbf{E}}, \quad \mathbf{C}=4\,\frac{\partial^2\psi}{\partial\mathbf{C}\partial\mathbf{C}}=\frac{\partial^2\psi}{\partial\mathbf{E}\partial\mathbf{E}}, \tag{2.7}$$

and Kirchhoff's stress tensor $\boldsymbol{\tau}=\mathbf{FSF}^{\mathbf{T}}$ and elastic stiffness tensor in spatial description

$$\boldsymbol{\tau}=2\mathbf{F}\,\frac{\partial\psi}{\partial\mathbf{C}}\mathbf{F}^{\mathbf{T}}, \quad \boldsymbol{c}=(\mathbf{F}\otimes\mathbf{F}^{\mathbf{T}}):\frac{\partial^2\psi}{\partial\mathbf{C}\partial\mathbf{C}}:(\mathbf{F}^{\mathbf{T}}\otimes\mathbf{F}), \tag{2.8}$$

where elastic modulus in spatial description $\boldsymbol{c}$ is related to elastic modulus in material description $\mathbf{C}$ as follows:

$$c^{abcd}=F^a_A F^b_B F^c_C F^d_D C^{ABCD}. \tag{2.9}$$

2.1. Material reinforced by one family of fibres

If we consider the material as transversely isotropic relative to undistorted state in fact we consider materials in which in the reference configuration possess a single preferred direction at each particle, although this need not be the same direction for every particle. Thus, constitutive equations are invariant under rotations about the preferred direction. For convenience of description, we employ terminology, as given by Spencer [1], which is appropriate for a material which is reinforced by a single family of continuously distributed fibres, although the theory is applicable to any material with a single preferred direction.

The fibre direction is defined by a unit vector field in the reference configuration which is denoted by $\mathbf{a}_0$ and is a function of the reference position $\mathbf{X}$, whose trajectories are termed fibres. In a deformation the fibres are conveying with the material, so that in a deformed configuration the fibre direction is defined by a unit vector field $\mathbf{a}$, where the stretch in fibre direction λ_a and its quadratic may be calculated as follows:

$$\lambda_a\mathbf{a}=\mathbf{F}\mathbf{a}_{0,} \quad \text{and} \quad \lambda_a^2=\mathbf{a}_0\mathbf{C}\mathbf{a}_{\mathbf{0}}. \tag{2.10}$$

For transversely isotropic material the strain energy function is invariant under arbitrary rotation of the reference configuration about the direction $\mathbf{a_0}$. It can be shown that that transversal isotropy requires strain energy function to be an isotropic invariant of $\mathbf{C}$ and $\mathbf{a_0}$, or alternatively $\mathbf{a}$. Taking into account that the

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