International Journal of Non-Linear Mechanics 73 (2015) 100–107

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Rigid body coupled rotation around no intersecting axes

Ljiljana Veljović [a,*], Aleksandar Radaković [b], Dragan Milosavljević [a], Gordana Bogdanović [a]

[a] Faculty of Engineering, University of Kragujevac, Sestre Janjic 6, 34000 Kragujevac, Serbia
[b] The State Universitz of Novi Pazar, Vuka KaradyicaKaradzica b, 36300 Novi Pazar, Serbia

ARTICLE INFO

Article history:
Received 18 March 2014
Received in revised form
28 August 2014
Accepted 4 November 2014
Available online 26 November 2014

Keywords:
Coupled rotations
No intersecting axes of rotation
Mass moment vectors
Phase trajectory
Angle of inclination
Graphical presentations

ABSTRACT

In this paper rigid body dynamic with coupled rotation around axes that are not intersecting is described by vectors connected to the pole and the axis. These mass moment vectors are defined by K. Hedrih. Dynamic equilibrium of rigid body dynamics with coupled rotations is described by vector equations. Also, they are used for obtaining differential equations to the rotor dynamics. In the case where one component of rotation is programmed by constant angular velocity, the non-linear differential equation of the system dynamics in the gravitational field is obtained and so is the corresponding equation of the phase trajectory. Series of phase trajectory transformations in relation with changes of some parameters of rigid body are presented.

1. Introduction

Non-linear oscillation systems are a phenomenon that occurs in all areas of science. Non-linearity exists in the phenomena in physics, chemical reactions and biology, as well as in the economy and medicine and society. Back in 1911 M. Petrovic wrote about founding mathematical theory that would describe phenomena that act similarly although they do not have the same cause [1]. For describing this phenomenon it is not necessary to go into its nature. Some mathematicians, who are not engaged in the experiment, do not have to know the cause of the phenomenon. Before the existence until the arrival of computers, the solution of some differential equations was obtained from analog electrical circuit, velocities and distances were defined by measuring electrical current. The equivalent electrical circuit is obtained by comparing the equations of motion for both systems.

Many engineering components consist of vibrating systems that can be modeled using oscillator systems. Every spinning rotor, because it is impossible to make it perfectly mass balanced, has some vibrations. Vibration of gear-pairs, for instance, was studied over the years. Machine tools, gear-pair system, shafts and computer disk drives are some real systems where minimizations, not total eliminations of unwanted vibrations have to be done. Aerospace vehicles, bridges, and automobiles are examples of structures for which many aspects have to be taken into consideration in the design to improve their performance and extend their life. So, it can be said that the problem of rigid body rotation has been studied over several centuries. With new operating conditions and growing old machines in the industry rotor dynamic research in this area is becoming more and more important. The need for reliable operations of the machines is of worldwide importance. It is important that dynamical questions, regarding the cause of the failures, can be identified and answered. The control of these vibrations in mechanical systems is still a flourishing research field as it enables resistance improvement as well as noise reduction.

First studying on rotating machines dates from Rankin's paper [2] in 1869. In 1919, Jeffcott created the idealized rotor model with the disc in the middle of the shaft which includes both vertical and horizontal motion of the rotor [3]. Today, in rotor dynamical systems with multi-degree of freedom rotors modeled as beams are sectioned applying finite elements and standard finite-element procedure.

Very old engineering problem which dates from 100 years ago is dynamics of coupled rotations. The dynamics of such systems are interesting and complex. Some essential results on the dynamics of coupled harmonic oscillators are obtained by consideration of two identical free oscillators mutually coupled [4]. The target is to determine when an unexpected vibration and in some cases failure can occur and therefore find out the causes of the problems. So, it is necessary to investigate properties of non-linear dynamics. The dynamics of coupled rotations is governed by linear and non-linear differential equations. The solution of linear problem is much easier and often it is sufficient for engineering work. The fact is that all systems have some non-linearity. Sometimes the best linear models could not portray all the vibration features that might be obtained from vibration measurements on actual machinery. The behavior of non-linear systems is much

* Corresponding author. Tel.: +381 648626500.
E-mail address: veljovicljilja@yahoo.co.uk (L. Veljović).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.11.001
0020-7462/

richer than that of linear systems. It is difficult to find the exact or closed-form solutions for non-linear problems. A systematic classification of the solutions based on the structure of differential equations is not generally possible. Non-linear analysis of complex systems is one of the most important and complicated tasks, especially in engineering and applied sciences problems. There are many non-linear equations in the study of different branches of science which do not have analytical solutions. So, many analytical and numerical approaches have been investigated. The solution to a problem depends largely on the researchers, their interests and areas of researches. For solving non-linear vibration problems many different methods may be used. For high-speed rotor symmetrically supported on the magneto-hydrodynamic bearing instability regions and amplitude level contours of vertical and horizontal vibrations are obtained in the frequency–amplitude plane [5]. It is a form suitable for engineering applications.

Combination of Newton's method and the harmonic balance method presented in [6] led to new technique for solving large amplitude oscillations of a class of conservative single degree-of-freedom systems with odd non-linearity. Instead original non-linear systems, the systems of linear equations are obtained. This technique, basically quite simple, does not require the presence of small perturbation parameter. In the paper [7], the analytical averaging method is applied for analyzing of a system of three coupled differential equations which describe the motion of a shear-building portal plane frame foundation that supports an unbalanced direct current motor with limited power supply. Yia in [8] analyzed the longitudinal coupled vibrations in a flexible shaft with multiple flexible disks. The analysis rotor model consists of multiple flexible disks attached to a flexible shaft with varying annular cross-section, as used for computer storage devices or steam turbines. The effects of disk flexibility on the longitudinal coupled vibrations between the shaft and disks are investigated with varying spindle rotational speed.

The mathematical equations which can be used to analyze vibrations in mechanical systems must be obtained primarily. In this paper mass moment vectors are used to present vector method for obtaining non-linear differential equations and the analysis of kinetic parameter of dynamics of coupled rotations. The definitions of mass moment vectors coupled to the pole and the axis are basic for the vector method. Mass moment vectors are presented and analyzed in numerous papers by Hedrih [9–12]. The phenomenon of appearance and disappearance of a trigger of coupled singularities and homoclinic orbits in the form of number eight in the phase portrait in the phase plane is investigated by using example of non-linear dynamics of a pair of coupled gears [13]. That phenomenon is an accompanying phenomenon of loss of stability of the local unique equilibrium position. Based on the introduced vector method and mass moment vectors non-linear phenomena in rotor dynamics were investigated in the series of references and presented in papers [11–16]. These non-linear phenomena include phase portraits and homoclinic orbits visualization of non-linear dynamics of multiple step multipliers, non-linear dynamics of planetary reductors with turbulent damping, non-linear dynamics of a heavy material particle along a circle which rotates, and homoclinic orbit layering in the coupled rotor non-linear dynamics and the chaotic clock models. The vector expressions of kinetic parameters of a rigid body simple or coupled rotation around no intersecting axes are derived and detailed explained in [9–12].

2. Model of a rigid body coupled rotations around two no intersecting axes

The principal vector is $\vec{\mathrm{J}}^{(O)}_{\vec{n}}$ of the body mass inertia moment at point O for the axis oriented by the unit vector $\vec{n}$ as $\vec{\mathrm{J}}^{(O)}_{\vec{n}} \stackrel{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]dm$, and there is a corresponding $\vec{\mathrm{D}}_O^{(\vec{n})}$ vector of the rotational rigid body mass deviational moment for the rotation axis $\vec{\mathrm{D}}^{(O)}_{\vec{n}} = \left[\vec{n},\left[\vec{\mathrm{J}}^{(O)}_{\vec{n}},\vec{n}\right]\right]$ defined by Hedrih [10].

The body mass inertia moment tensor $\vec{\mathrm{J}}^{(O)}$ for point O is determined with six scalar dynamic parameters

$$\vec{\mathrm{J}}^{(O)} = \begin{bmatrix} J_u & D_{uv} & D_{un} \\ D_{vu} & J_v & D_{vn} \\ D_{nu} & D_{nv} & J_n \end{bmatrix}$$

and it is analogous to the stress tensor and strain tensor in the elasticity theory [13]. In the case when rigid body is balanced with respect to the axis the mass inertia moment vector $\vec{\mathrm{J}}^{(O)}_{\vec{n}}$ is collinear to the axis, axis of rotation is main axis of body inertia. When axis of rotation is not main axis, then the rotor is unbalanced to the axis and the mass inertial moment for the axis contains deviation part $\vec{\mathrm{D}}^{(O)}_{\vec{n}}$ [11].

The corresponding body mass linear moment of the rigid body at point O for the axes oriented by the unit vector $\vec{n}$ is $\vec{\mathrm{S}}^{(O)}_{\vec{n}} = \iiint_V \left[\vec{n},\vec{\rho}\right]dm$. Here $\vec{\rho}$ is vector position of mass dm, $\vec{n}$ is axis unit vector.

The rigid body coupled rotations around two no intersecting axes is first presented in [11]. The first axis, fixed positioned, is oriented by unit vector $\vec{n}_1$. The second axis is oriented by unit vector $\vec{n}_2$ which is rotating around fixed axis with angular velocity $\vec{\omega}_1 = \omega_1 \vec{n}_1$. Rigid body is positioned on the moving rotating axis oriented by unit vector $\vec{n}_2$. Rigid body rotates around rotating axis with angular velocity $\vec{\omega}_2 = \omega_2 \vec{n}_2$ and around fixed axis oriented by unit vector $\vec{n}_1$ with angular velocity $\vec{\omega}_1 = \omega_1 \vec{n}_1$, so its angular velocity is $\vec{\omega} = \omega_1 \vec{n}_1 + \omega_2 \vec{n}_2$. The rigid body is screw (inclined) positioned on the self-rotation axis. The angle β is angle of screw position of rigid body to the self-rotation axis. When center C of the mass of rigid body is not on self-rotation axis of rigid body self-rotation, it is said that rigid body is eccentrically positioned in relation to the self-rotation axis. Eccentricity of position is normal distance between body mass center C and axis of self-rotation and it is defined by $\vec{e} = \left[\vec{n}_2,\left[\vec{\rho}_C,\vec{n}_2\right]\right]$. Here $\vec{\rho}_C$ is vector position of mass center C with origin in point O_2, and position vector of mass center with fixed origin in point O_1 is $\vec{r}_C = \vec{r}_0 + \vec{\rho}_C$, $\vec{r}_0 = \overrightarrow{O_1O_2}$. For detail see Ref. [11] and Fig. 1.

3. Vector forms of the derivatives of linear and angular momentum of rigid body coupled rotations around two no intersecting axes

Vector expressions for linear and angular momentum are expressed in Ref. [12]. By using vector method based on the mass moment vectors and vector rotators for a rigid body coupled rotations around no intersecting axes, the vector expressions of linear and angular momentum are presented in the following form:

$$\vec{\mathrm{K}} = \left[\vec{\omega}_1,\vec{r}_0\right]M + \omega_1 \vec{\mathrm{S}}^{(O_2)}_{\vec{n}_1} + \omega_2 \vec{\mathrm{S}}^{(O_2)}_{\vec{n}_2} \quad (1)$$

$$\vec{\mathrm{L}}_{O_1} = \omega_1\left[\vec{r}_O,\left[\vec{n}_1,\vec{r}_O\right]\right]M + \omega_2\left[\vec{r}_O,\vec{\mathrm{S}}^{(O_2)}_{\vec{n}_2}\right] + \omega_1\left[\vec{r}_O,\vec{\mathrm{S}}^{(O_2)}_{\vec{n}_1}\right] + \omega_1\left[\vec{\rho}_C,\left[\vec{n}_1,\vec{r}_O\right]\right]M + \omega_1 \vec{\mathrm{J}}^{(O_2)}_{\vec{n}_1} + \omega_2 \vec{\mathrm{J}}^{(O_2)}_{\vec{n}_2}. \quad (2)$$