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Vibration analysis of beams with multi-layers of corrugations

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ABSTRACT

In this paper, an impedance method for the free vibration analysis of beams with multi-layers of corrugations is introduced. In particular, the vibration response of a beam with layers of corrugations is formulated using the impedance method. The corrugated beam is divided into two kinds of structural segments: one, the corrugation that is modeled as a curved beam and the other, the liner treated as a straight beam. The frequency equation is derived by assembling the impedance of each structure segment based on conditions of force equilibrium and velocity compatibility. The accuracy of the proposed modeling is accessed using numerical examples analyzed both with the proposed method and the finite element commercial software ANSYS. A good agreement between the results for the proposed method and those from ANSYS analysis is observed. Then the influences of the number of corrugated layers and the corrugation number in each layer on the beam bending stiffness are investigated.

1. Introduction

For a fixed volume of mass, the corrugation becomes a good choice in distributing the mass away from the neutral axis to increase the effective height of a structure, which results in a larger area moment of inertia and hence the greater bending stiffness. Therefore as compared to those without corrugations, the corrugated structure enjoys a better strength/weight ratio. Moreover, corrugations on structures can be easily created in a cost-effective manufacturing process; hence, flat plates or shells reinforced using corrugations become popular and can be found in structures such as sandwich plate cores, roofing, decking, packing boxes, etc. Fig. 1 illustrates a typical corrugated plate which consists of the liner and the curvilinear shapes, i.e., the corrugation or called medium. Note that the corrugation lying in different planes makes the mechanical analysis very complicated. Traditionally an analytical analysis is performed using equivalent rigidity through an approach to approximating the orthotropic plate. However, the accuracy of this approximated rigidity due to structures with complicated shape is questionable [1].

On the other hand, numerical methods enjoy great success in engineering and the finite element method becomes the most popular technique in performing the mechanical analysis for structures with complicated geometry. For structures having geometry with discontinuity, i.e., cracks, the computation using the finite element method becomes intensive and time consuming due to great number of elements required near the geometric discontinuity. To solve this difficulty caused by element meshing, the mesh-free Galerkin method proves to be an alternative and Liw etc. revealed its effectiveness in analyzing the corrugated plates [2–4]. Nevertheless, the mesh-free Galerkin method becomes much more complex if the plate consists of multiple corrugated layers.

To address the issue of vibration analysis of structures with multiple layers of corrugations, an impedance method that fully utilizes the geometric features in periodicity of multi-layer corrugated structures is introduced.

The impedance method specifically aims for solving the dynamical system from the point of view of “system”. The input-output relation for a dynamic system is explicitly expressed using the transfer function, called mobility or its inverse, the impedance which can be obtained in many ways; analytically, numerically or experimentally [5–11]. When the analytical model of a structure is not available due to complicated geometry or boundary conditions, the experimental measurement (*e.g.*, impact testing) or numerical simulation (*e.g.*, finite element method) is adopted to find the mobility of a structure. Moreover, the impedance method provides a useful method in investigating the dynamical behavior of one portion of a mechanical system independently with respect to the rest of the system. That is, one may break up a complicated system into substructures with simple dynamic characteristics. Then the transfer function of the whole system can be integrated based on displacement compatibility and force equilibrium at the interface (see *e.g.*, Bishop and Johnson [5]). Although similar method, *e.g.* the transfer-matrix method can also be found in [12]; however, exploiting the advantage of this semi-analytical

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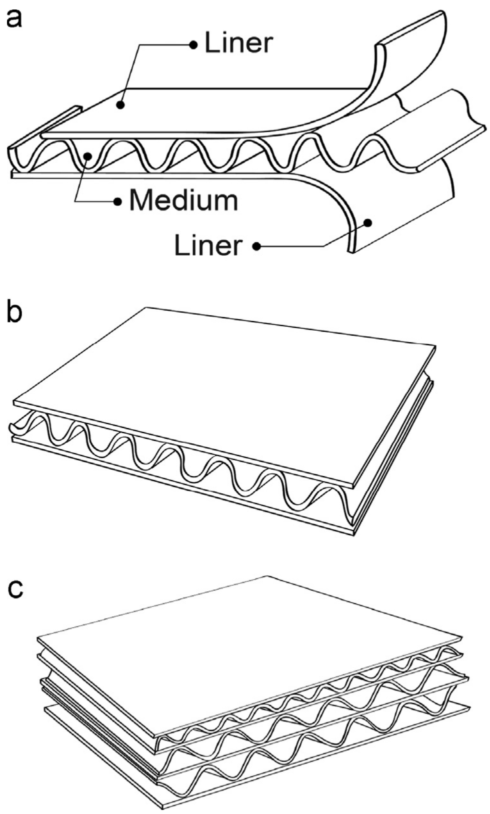

Fig. 1. (a) Compositions of corrugated plates; (b) single layer of corrugated plate; and (c) triple layers of corrugated plate.

method in modeling corrugated structures, to author's knowledge has not been performed.

For a structure involving large amount of degree of freedoms, how to improve the computational efficiency without sacrificing the related accuracy is always an issue to be addressed. In dynamic problems, the substructure method of finite element method, which reduces the number of equations to manageable size by dividing the original structure into several substructures and then treating each substructure as a large element, i.e. super element with boundary nodes after static and dynamic condensations is a commonly used technique. Nevertheless, it is worthy of noting that, techniques such as static and dynamic condensations e.g. the Guyan reduction [13,14], the improved reduced system (IRS) [15] and the iterated IRS reduction [16,17] which reduce the order of the original structural model by removing some degrees of freedoms from the original finite element model do not have an accurate simulation because of the approximation in their formulations. For example, the Guyan reduction is limited to the static analysis because only the stiffness is considered in the modeling. And the IRS, iterated IRS reduction and the modified reduction methods which take the inertia effect into consideration extend the model condensation method to the dynamic problems [18–21]. However, those methods do not provide accurate prediction at high frequencies due to neglect of high order inertia terms in the approximation. On the other hand, the dynamic characteristics of substructures represented by the impedance can be obtained in many ways; analytically, numerically or experimentally. Moreover, the assembly of the subsystems using the impedance method can also be considered as an alternative of the condensation method; however, without any approximation involved in the formulation. Consequently, using the impedance method in dynamic analysis of structures is more accurate than common substructure methods.

In this paper, the corrugated beam is deemed as a periodic structure with liner and medium as two basic substructures. The impedance of a corrugation modeled as a curved beam can be obtained using the finite element method, whereas the impedance of a liner deemed as a straight beam can be expressed analytically. An assemble scheme that fully utilizes the feature in structural periodicity of corrugated structures is proposed to integrate the substructures into a multiple-layer corrugated beam. Then free vibration analysis of corrugated beam is presented and the result is compared with that from commercial finite element analysis package ANSYS for validation. Furthermore the influences of the number of layers of corrugations and the corrugation number in each layer on the bending stiffness of beam are investigated to demonstrate the effectiveness of this proposed method.

2. Impedance couplings between structural subsystems

Consider a two dimensional physical model of corrugated beam composed of liners and mediums as shown in Fig. 2. Assume that the liner is approximated as a straight beam, whereas the medium is made of corrugations modeled as curved beams connected in series. That is, the corrugated beam, namely, is divided into curved and straight structural modules as Fig. 2(b) illustrated. Each module is deemed as a structural subsystem. The dynamic response of corrugated beams can be determined if the dynamic characteristic of each module is obtained. Fig. 3(a) depicts several

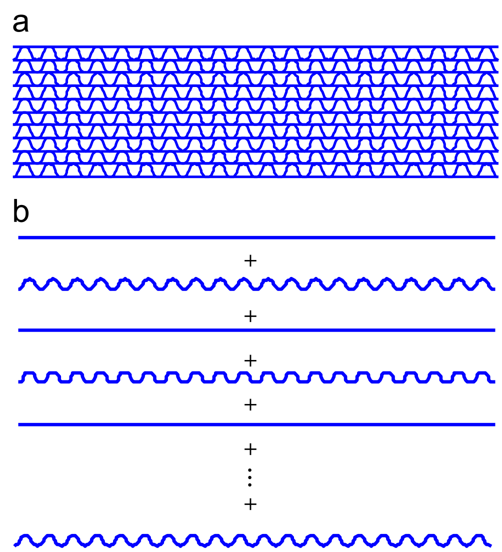

Fig. 2. Modular modeling corrugated beam; (a) corrugated beam with ten layers of corrugations and (b) subdivision of corrugated beam.

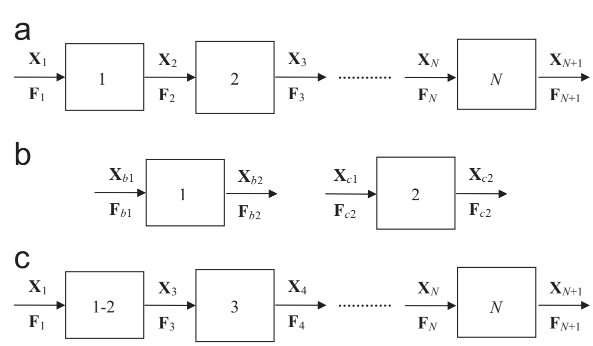

Fig. 3. The methodology in coupling subsystems; (a) the structure is divided into N subsystem; (b) coupling between two subsystems; and (c) coupling subsystem 1–2 to the rest of subsystems.