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Effects of different shear deformation theories on free vibration of functionally graded beams

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ABSTRACT

Free vibration of functionally graded (FG) beams subjected to different sets of boundary conditions is examined in the present paper. Different higher-order shear deformation beam theories (SDBTs) have been incorporated for the free vibration response of FG beam. The material properties of FG beam are taken in the thickness direction in power-law form and trial functions denoting the displacement components are expressed in algebraic polynomials. The Rayleigh–Ritz method is used to estimate frequency parameters in order to handle to various sets of boundary conditions at the edges in a simple way. Comparison of frequency parameters is carried out with the existing literature in special cases and new results are also provided after checking the convergence of frequency parameters.

1. Introduction

The present investigation is associated with the use of the Rayleigh–Ritz method in free vibration response of functionally graded beams. The Rayleigh–Ritz method (after Lord Rayleigh and Walther Ritz) is an approximate computational technique which is extensively used in several research areas. A brief idea about this method can be available in [1–4]. Functionally graded materials (FGMs) have been widely used in most of the industrial applications and structural engineering design viz. aerospace, nuclear, biomedical, electronics and in many other fields. Concept of FGMs was first enunciated in 1984 by a group of material scientists while preparing a space-plane project in Japan [5]. FGMs are the special composite materials that have been developed because of their high temperature-resistant properties through a comparatively less thickness. The primary constituents for these materials are metal with ceramic or from a combination of materials. The ceramic constituent provides high-temperature resistance due to its low thermal conductivity. The ductile metal constituent on the other hand prevents fracture caused by stresses due to high temperature gradient in a very short span of time. The material properties in FGMs vary continuously in the thickness direction in power-law exponent form. The present literature reveals the works done towards the analysis of FGMs by different researchers throughout the globe.

Consequently, computational (numerical) techniques to analyze FGMs are also in huge demand in research sectors day-by-day. Chakraborty et al. [6] proposed a new beam finite element based on the first-order shear deformation theory to study the thermo-elastic behavior of functionally graded beam structures. Shahba et al. [7] investigated free vibration and stability analysis of axially functionally graded Timoshenko tapered beams using classical and non-classical boundary conditions through finite element approach. Ruocco and Minutolo [8] have presented a field boundary element model to solve elastic functionally graded materials for two-dimensional stress analysis. A new approach has been employed by Huang et al. [9] for investigating the vibration behaviors of axially functionally graded beams with non-uniform cross-section. Free bending vibration of rotating functionally graded Euler–Bernoulli tapered beams with different boundary conditions is investigated in [10] using the differential transform method and the differential quadrature element method. An improved third-order shear deformation theory is employed to check thermal bucking and elastic vibration of functionally graded beams [11].

As such, different researchers throughout the globe have implemented various different SDBTs to estimate vibration response of functionally graded structural beams. Aydogdu and Taskin [12] studied the free vibration behavior of a simply supported FG beam by using Euler–Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. A new beam theory was considered by Sina et al. [13] different from traditional first-order shear deformation beam theory to analyze the free vibration of functionally graded beams with an analytical approach. Şimşek [14] has examined vibration response of a simply supported FG beam to a moving mass by using Euler–Bernoulli, Timoshenko and the third-order shear deformation beam theories. Using different higher-order shear deforma-

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tion beam theories, Şimşek [15] has also recently studied the fundamental frequencies of FG beams subjected to different boundary conditions. Alshorbagy et al. [16] have used the finite element method to detect the free vibration characteristics of a functionally graded beam. Recently, free vibration and stability of axially functionally graded tapered Euler–Bernoulli beams are investigated using the finite element method by Shahba and Rajasekaran [17]. Using the analytical method, Thai and Vo [18] have developed bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories.

One may also see the use of the Rayleigh–Ritz and the Ritz method to analyze vibration behavior of isotropic as well as FG structural members. The Rayleigh–Ritz method (after Walther Ritz and Lord Rayleigh) is an approximate numerical method to study the natural vibration frequencies of different types of structural members. The characteristic orthogonal polynomials in the Rayleigh–Ritz method were used by Bhat [19] to estimate the transverse vibration response of rotating cantilever beam with a tip mass. In another literature by Bhat [20] computed the natural frequencies of rectangular plates using characteristic orthogonal polynomials in the Rayleigh–Ritz method. Singh and Chakraverty [21–23] studied transverse vibration of elliptic and circular plates using orthogonal polynomials in the Rayleigh–Ritz method satisfying different boundary conditions viz. completely free, simply supported and clamped. Vibration response of cross-ply laminated beams has been explained in [24] and buckling analysis is done in [25] with general boundary conditions by the Ritz method. The plane stress problem of an orthotropic functionally graded beam with arbitrary graded material properties along the thickness direction is investigated recently by the displacement function approach by Nie et al. [26]. Pradhan and Chakraverty [27] have also applied the Rayleigh–Ritz method to free vibration of Euler and Timoshenko functionally graded beams subject to various boundary conditions. Vo et al. [28] have presented static and vibration analysis of FG beams using refined shear deformation theory by using finite element formulation. Most recently, static and free vibration of axially loaded rectangular FG beams is developed in [29] based on the first-order shear deformation beam theory. It is evident from the present literature that no detailed study is yet done using the Rayleigh–Ritz method to study free vibration of FGM beams with different shear deformation beam theories.

In view of the above, the objective is to develop a reliable and efficient computational modelling for the vibration behaviors of FGM beams subjected to different boundary conditions within the framework of various SDBTs mentioned above. The origin of the Cartesian co-ordinate system is spatially taken at the center of the FG beam. Modelling of this problem and the solution methodology using simple polynomial functions in the Rayleigh–Ritz method has been developed. Trial functions denoting the displacement fields in the subsequent numerical formulation part are expressed in simple algebraic polynomial forms, which will satisfy the essential boundary conditions for the ease of computation. Various SDBTs viz. classical beam theory (CBT), Timoshenko beam theory (TBT), parabolic shear deformation beam theory (PSDBT), exponential shear deformation beam theory (ESDBT), trigonometric shear deformation beam theory (TSDBT), hyperbolic shear deformation beam theory (HSDBT) and a new shear deformation beam theory (ASDBT) are demonstrated here to define the displacement components. Results from our study are compared with those obtained from literatures available and are found to be in good agreement. New results for free vibration of FG beam subjected to different sets of boundary conditions (BCs) viz. Clamped–Clamped (C–C), Simply supported–Simply supported (S–S) and Clamped–Free (C–F), are also obtained and hence reported. It is believed that the tabulated results will probably help other researchers to compare their results related to these problems.

2. Functionally graded materials

A straight FG beam of length L, width b and thickness h, having rectangular cross-section with Cartesian coordinate system O (x, y, z) and having the origin at O is shown in Fig. 1.

It is assumed that the material properties of FG beam vary along the thickness direction according to power-law form as shown in Fig. 2. The power-law variation used in [13] is considered

$$P(z) = (P_c - P_m)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m \tag{1}$$

where P_c and P_m denote the values of the material properties of the ceramic and metal constituents of the FG beam respectively. k (power-law exponent) is a non-negative variable parameter. According to this distribution, the bottom surface $(z = -h/2)$ of FG beam is pure metal, whereas the top surface $(z = h/2)$ is pure ceramic and for different values of k one can obtain different volume fractions of material beam as mentioned in [12]. For our present formulations, the material properties viz. Young's modulus (E) and mass density (ρ) are taken to vary along the thickness direction except Poisson's ratio (ν) remaining as constant. In Fig. 2, FG constituents own the properties [13]: $E_m = 70$ GPa, $\rho_m = 2700$ kg/m^3, $E_c = 380$ GPa and $\rho_c = 3800$ kg/m^3.

3. Numerical modelling and formulation

Let us assume the deformation of functionally graded beam in the x–z plane and denote the displacement components along x, y and z directions by u_x, u_y and u_z respectively. Based on the higher order shear deformation beam theory, the axial displacement (u_x) and the transverse displacement (u_z) of any point of the beam are given in Eqs. (2) and (3) as below [12]

$$u_x(x, z) = u(x, t) - z w_{,x}(x, t) + \Phi(z) v(x, t) \tag{2}$$

$$u_z(x, z) = w(x, t) \tag{3}$$

where u and w represent the axial and the transverse displacement of any point on the neutral axis respectively, while v is an unknown function that represents the effect of transverse shear strain on the neutral axis. Φ represents the shape function determining the distribution of the transverse shear stress and strain through the thickness of the beam and $(\ )_{,x}$ indicates the derivative with respect to x. Different theories can be obtained by choosing their respective shape functions $\Phi(z)$. The present study is concerned with SDBTs viz. CBT, TBT, PSDBT, ESDBT, HSDBT, TSDBT and ASDBT, as mentioned in [15]. $\Phi(z)$ for these shear deformation beam theories are given in Eq. (4) as below [15]

$$\text{CBT}: \Phi(z) = 0$$
$$\text{TBT}: \Phi(z) = z$$
$$\text{PSDBT}: \Phi(z) = z\left(1 - \frac{4z^2}{3h^2}\right)$$
$$\text{ESDBT}: \Phi(z) = z e^{-2(z/h)^2}$$

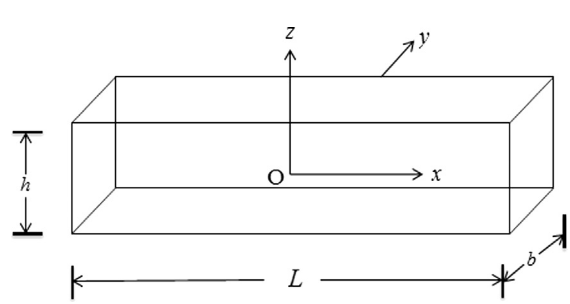

Fig. 1. A typical functionally graded beam element with Cartesian coordinates.