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Dynamic response of a cylinder cover under a moving load

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ABSTRACT

Cylinders with thin covers are used in high-speed rolling contact industrial applications such as a two-cylinder soft calender of a paper machine. In this paper, the dynamic behavior of an elastic cylinder cover is studied using a 1D Pasternak-type foundation model with Kelvin–Voigt damping. The cover is subjected to a moving point load, which is taken to represent a load resultant due to rolling contact. Analytical expressions for the natural frequencies, vibration response, wave dispersion relation, total strain energy and dissipation power of the cover are obtained. To validate the 1D approach, the calculated natural frequencies and modes are compared to those given by a 2D plane strain finite element model, and a good agreement is found. The critical load speed at which traveling waves first appear in the cover is derived for the undamped analytical model on the basis of a resonance condition. The critical speed is shown to be also the minimum phase velocity of the waves in the cover. When damping is included, the wave speeds decrease, lowering also slightly the critical speed, which, in addition, becomes blurred due to the damping. Once a traveling wave has emerged, it remains in the cover also at supercritical speeds due to a spectrum of resonant speeds induced by wave dispersion. At supercritical speeds, reinforced resonances are observed when the head and tail of a traveling wave interact. High shear damping leads to a substantial increase in dissipation power related to heat generation and rolling resistance of the cover already at subcritical speeds.

1. Introduction

Along with the advances in materials science, the development of cylinder cover materials for rolling contact industrial machines, for example soft calenders of paper machines, has taken major steps forward in recent years. The replacement of the traditional metal-to-metal contact by novel polymers and composites in high-speed rolling contact applications has proven to be beneficial in terms of end product quality and modifiability. However, the covers induce and suffer from detrimental dynamic phenomena which have not been fully explained yet, let alone dealt with. Examples of these phenomena are the self-excited vibration mechanism, barring, which is caused by viscoelastic cover deformations acting as time-delayed feedbacks in a rolling contact system [1,2], and a contact-induced traveling wave phenomenon occurring at high rolling speeds, which is the subject of this study.

The study of cylinder cover dynamics constitutes essentially a 2D plane strain problem where typically a relatively long hard cylinder with a thin soft cover is rolling in contact with another cylinder. It is crucial to note that the plane strain feature establishes a fundamental distinction between covered cylinders and, for example vehicle tires and flexible train wheels, which are often studied by using ring or shell models. In other words, the traveling waves in cylinder covers should not be viewed too closely in terms of other circular structures.

Only a few studies addressing traveling waves in thin cylinder covers can be found. In a recent paper, Qiu developed a 2D semianalytical plane strain model for a covered cylinder rolling in contact with a rigid surface [3]. He calculated the natural frequency spectrum for the covered cylinder and used the frequencies to estimate the critical rolling speed at which traveling waves start to emerge in a cover. To understand the nature of the traveling wave phenomenon to a deeper extent, we recently utilized a 2D plane strain finite element (FE) model to study a polymer-covered cylinder rolling in contact with a steel cylinder [4]. We found that at the critical speed of the covered cylinder, the minimum speed of Rayleigh waves in the cover is actually reached and both primary and secondary Rayleigh waves arise at the leading edge of the contact area. The superposition of the waves leads to the formation of a strong traveling wave in the cover. The phenomenon may be considered as a Rayleigh wave resonance [5]. A similar wave propagation phenomenon is encountered, for example, when a high-speed train approaches the Rayleigh wave speed of the train–track subsoil, see, for example, [6].

The detailed account on the physical characteristics of the traveling wave phenomenon in a thin cylinder cover under rolling

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contact [4] acts as a basis for the current study. The central novel feature of this paper is that we show how the dynamic 2D plane strain cover problem can be essentially reduced to 1D. The development of such a reduced model is motivated by the general notion that a 1D model should be more convenient to work with both analytically and computationally than a 2D model.

In more detail, we study traveling waves in a cylinder cover subjected to a moving constant radial point load using a 1D Pasternak-type foundation model with Kelvin–Voigt damping. The cover vibration is investigated using a modal expansion technique and the wave propagation offers a complementary perspective via wave dispersion analysis. The natural frequencies and modes calculated from the 1D analytical model are compared to those given by a 2D plane strain FE model to show that the 1D model captures the essential features of the 2D problem. A major outcome from the 1D approach is that it allows the effortless calculation of the relevant resonant load speeds, dispersion curves and vibration response unlike semianalytical [3] or purely numerical methods [4,7–10]. The transient cover response sheds light on the emergence of traveling waves in the cover. The calculated steady-state response demonstrates the effect of cover damping in a detailed manner, showing how the retardation of the response with respect to the moving load is explained by sub- and supercritical modes; how the traveling waves emerge in the cover for different values of cover damping; and how the damping affects the power dissipation of the cylinder cover at sub- and supercritical speeds.

2. Theory and physical interpretation

2.1. Equation of motion and vibration response

The system under investigation is depicted schematically in Fig. 1. The 1D model consists of a non-rotating rigid cylinder with a cover subjected to a moving radial point load P. In other words, an actual case of rolling contact is studied as an inverted problem [11]. The effect of the rotation of the cylinder will be discussed later, but it will turn out that it is not of crucial importance for the current study. In reality, the load would be a distributed one, but since the contact area, the nip, is typically relatively small in rolling contact machines, it is reasonable to present the load resultant as a point force acting at the load center. The cover is modeled as a Pasternak-type foundation consisting of a shear layer attached to a Kelvin–Voigt assembly.

When the traveling wave phenomenon takes place under rolling contact in a 2D system, the dominant modes of vibration are those of the primary mode family (see Fig. 20 in [4]). Furthermore, at such an instance, the displacement path of a point near the cover surface due

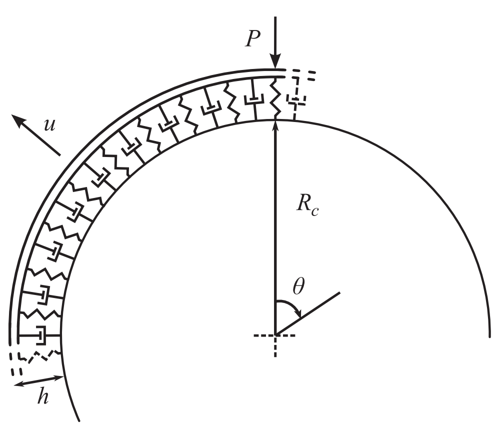

Fig. 1. 1D Analytical model for a covered cylinder subjected to circumferentially moving point load P. The cover is modeled as a shear layer attached to a Kelvin–Voigt base, and u is the radial displacement of the shear layer.

to a traveling wave (Rayleigh wave) is strongly elliptic with the radial displacements clearly dominating over the tangential ones. On the basis of the foregoing, a single-mode approximation is used for the radial deformation u of the shear layer in the 1D model to capture the primary mode family, but the tangential deformations are not taken into account. It will be shown in Section 3.1 that this is a good approximation of the primary mode family of a 2D model.

The model in Fig. 1 is quite similar to the one by Chatterjee et al. [12]. Their treatment, however, was largely different from ours. They used their model to study waves in rotating tires by formulating a non-linear boundary value contact problem associated with steady-state rolling conditions. We employ a circumferentially moving point load catching the essentials of the traveling wave phenomenon in the cylinder cover. In the following, the complete vibration response is derived for the system of Fig. 1.

The equation of motion in a coordinate system fixed to the cylinder in terms of the radial displacement u of the shear layer reads

$$u_{tt}+\frac{E}{\rho h^2}u+\frac{\alpha}{\rho h^2}u_t-\frac{\kappa G}{\rho R^2}u_{\theta\theta}-\frac{\kappa\beta}{\rho R^2}u_{\theta\theta t}=P(\theta,t). \tag{1}$$

Above, E is Young's modulus and G is the shear modulus of the cover, and α and β are the corresponding strain rate damping parameters, respectively. The density and thickness of the cover are ρ and h, respectively. The radius $R=R_c+h/2$ is used to determine the effective width of a material element. In addition, we introduce a shape factor κ for the shear layer, which accounts for the total shear force on the cover layer cross-section similarly to the shear coefficient of a Timoshenko beam. Thus, in principle, the shape factor introduces a 2D effect to the model. The value of the shape factor is determined computationally in Section 3.1 with the aid of a 2D plane strain FE model. For a moving constant point load, we have $P(\theta,t)=P_0\delta(\theta-\Omega t)$, where P_0 is the load amplitude and Ω is the angular velocity of the load. The rotational frequency of the load is $f_{\text{rot}}=\Omega/2\pi$. The model description is completed by the requirement of continuity of the displacement and slope which leads to

$$u(0,t)=u(2\pi,t)\quad\text{and}\quad u_\theta(0,t)=u_\theta(2\pi,t). \tag{2}$$

Note that the term containing $u_{\theta\theta}$ in Eq. (1) stems from the Pasternak foundation model and is typical for all shear layer models. This term couples the adjacent material elements to each other. For a derivation of the Pasternak foundation model, see [13], by the aid of which the dynamic equation of motion Eq. (1) can be easily derived by taking into account the mass of the cover lumped to the shear layer.

Each natural mode, $\sin(n\theta)$ or $\cos(n\theta)$, of the free undamped cover consists of n full waves on the cover circumference, with the exception of $n=0$, which is the breathing mode. As n increases, the wavelength in a mode decreases. The solution for the moving load problem can be expanded in terms of the natural modes leading to

$$u(\theta,t)=\sum_{n=1}^{\infty}[c_n(t)\ \sin(n\theta)+d_n(t)\ \cos(n\theta)]+d_0(t). \tag{3}$$

By substituting Eq. (3) into Eq. (1), the modal expansion coefficients c_n, d_n and d_0 can be calculated from the equations

$$\ddot{c}_n+2\zeta_n\omega_n\dot{c}_n+\omega_n^2c_n=\frac{P_0}{\pi}\sin(n\Omega t), \tag{4}$$

$$\ddot{d}_n+2\zeta_n\omega_n\dot{d}_n+\omega_n^2d_n=\frac{P_0}{\pi}\cos(n\Omega t), \tag{5}$$

$$\ddot{d}_0+2\zeta_0\omega_0\dot{d}_0+\omega_0^2d_0=\frac{P_0}{2\pi}, \tag{6}$$