International Journal of Non-Linear Mechanics 71 (2015) 30–38

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Rotary motion of the parametric and planar pendulum under stochastic wave excitation

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ARTICLE INFO

Article history:
Received 3 June 2013
Received in revised form
3 December 2014
Accepted 18 December 2014
Available online 14 January 2015

Keywords:
Parametric and planar pendulums
Rotations
Simulation of random processes
Stochastic excitation
Rotational number

ABSTRACT

In this paper a rotary motion of a pendulum subjected to a parametric and planar excitation of its pivot mimicking random nature of sea waves has been studied. The vertical motion of the sea surface has been modelled and simulated as a stochastic process, based on the Shinozuka approach and using the spectral representation of the sea state proposed by Pierson–Moskowitz model. It has been investigated how the number of wave frequency components used in the simulation can be reduced without the loss of accuracy and how the model relates to the real data. The generated stochastic wave has been used as an excitation to the pendulum system in numerical and experimental studies. For the first time, the rotary response of a pendulum under stochastic wave excitation has been studied. The rotational number has been used for statistical analysis of the results in the numerical and experimental studies. It has been demonstrated how the forcing arrangement affects the probability of rotation of the parametric pendulum.

1. Introduction

The dynamics of the parametric pendulum has been the subject of study for many years, because of the interesting and rich dynamical behaviour exhibited by it [3,2,19,5]. It has been also one of the most celebrated examples studied in the literature to illustrate the dynamics of a non-linear oscillator. Number of studies deals with the oscillatory motion of the pendulum subjected to harmonic parametric excitation. The motion is analysed analytically by the multiple scales method by Xu and Wiercigroch [22], Mann and Koplov [16] and by the harmonic balance method by Sofroniou and Bishop [19]. A wider range of the responses can be observed once the pendulum escapes the separatrix and rotational motion of the pendulum in this range is of interest. Approximating the escape zone has been the topic of study for Trueba et al. [21], Thompson [20] and Bishop and Clifford [5] who used symbolic dynamics approach in their work. The rotational motion of the parametrically excited pendulum has become the topic of interest in recent years because of its potential for utilization in energy harvesting. Different types of rotations have been classified by Garira and Bishop [7] and Clifford and Bishop [4]. Limit of existence of rotational motion has been determined analytically by Koch and Leven [14] and Lenci at al. [15], who derived analytical approximation for the rotational motion, studied their stability using perturbation method and compared their analytical with numerical results. Xu and Wiercigroch [22] also derived analytical approximate solution for rotational motion using multiple scales method. Recently synchronized rotations of a system of pendulums are studied by Czolczynski et al. [6].

The interest in the study of the rotational motion of the parametric pendulum arises from the fact that it can be utilized for energy harvesting purposes. With this aim Xu et al. [23] have studied the rotational motion of the harmonically excited pendulum in great detail, analytically, numerically and experimentally. They identified the regions in the parametric plane of the frequency and amplitude of the harmonic excitation in which the rotational motion is obtained. The dynamics of the pendulum can become more complex depending on the type of excitation. Ge et al. [9,8] investigated the chaotic motion of a pendulum subjected to both rotation and vertical support vibration. The general insight into dynamics of the pendulum under different types of harmonic excitation has been presented by Trueba et al. [21]. Horton [11] observed that the introduction of a small horizontal motion in addition to the vertical excitation would significantly influence the stable periodic rotation region of the pendulum. Wiercigroch and Horton [12] studied the effects of the heave excitation on the rotation motion of the pendulum from the point of view of energy harvesting from sea wave excitation.

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http://dx.doi.org/10.1016/j.ijnonlinmec.2014.12.008
0020-7462/

Almost all the works in the literature on the rotational motion of the pendulum deal with harmonic parametric excitation. When one considers ocean wave excitation on the pendulum system for wave energy harvesting, it has to be treated as a random process. The response of the pendulum subjected to stochastic excitation has only recently been studied numerically by Yurchenko et al. [24]. Authors identified the regions with highest and lowest probability of rotation in the forcing parameter space, for the parametric pendulum under noisy excitation, where the noise has been modelled as a narrow-band process.

This paper is concerned with the numerical and experimental investigation of the rotational motion of the pendulum under specific type of random parametric excitation. The random excitation is assumed to arise from ocean wave excitation, which is modelled as a stationary random process having the Power Spectral Density (PSD) function of the Pierson–Moskowitz (PM) form [17]. The parameters of the PM spectrum are obtained to fit real wave data. The histories of the ocean waves are generated corresponding to this PSD using the Shinozuka method [18] of simulating random process as a sum of sine waves with different frequencies and random phase angles. Variable frequency increments are used based on equal spectral content within adjacent frequency ranges which facilitates the use of less terms in the sine series which reduces the time of simulation at the same time preserving the accuracy of simulation. The non-linear equation of motion of the pendulum is numerically integrated with the simulated time series and the rotational motion of the pendulum is investigated under the influence of random excitation. The numerical results are verified with experimental results performed on the laboratory pendulum set up with the random parametric excitation corresponding to the simulated time series scaled appropriately. The numerical results agree well with experimental results. The rotational motion under random parametric excitation is further characterized by rotational number.

The paper is organized as follows. Section 1 gave a brief introduction and a review of the current state of research on the rotational motion of the parametrically driven pendulum. In Section 2 a mathematical model for the pendulum with a random excitation is presented. Section 3 presents a stochastic model for the random wave excitation and the simulation of the random waves corresponding to the PM spectrum. In Section 4 the real sea wave data is analysed and Section 5 presents numerical and experimental results of the rotational motion of the pendulum subjected to random excitation.

2. Parametric pendulum modelling

The system studied in this paper is a parametric pendulum subjected to the stochastic wave excitation at its pivot in horizontal and vertical or only in the vertical direction. The two forcing arrangements are schematically represented in Fig. 1. The generalized equation of motion of a parametric pendulum forced on the plane is given by

$$ml\ddot{\theta}+m\ddot{X}\cos\theta+m(\ddot{Y}+g)\sin\theta+cl\dot{\theta}=0, \tag{1}$$

where X, Y can be arbitrary functions and if $X=0$ pendulum experiences only vertical forcing. For the classical parametric pendulum case studied in [2,19,5,22,16,4,15] X, Y are given by a harmonic function. Here they are the functions of the wave displacement and can be derived from the wave displacement function $f(t)$ containing reconstructed time history of the wave, applied at the pendulum support. The expressions for X and Y have been obtained from the kinematic relationships governed by a pendulum set-up geometry sketched schematically in Fig. 2. To introduce the forcing component in the horizontal direction the pendulum has been fixed with the parallel guide arms, causing the vertically excited support to move on

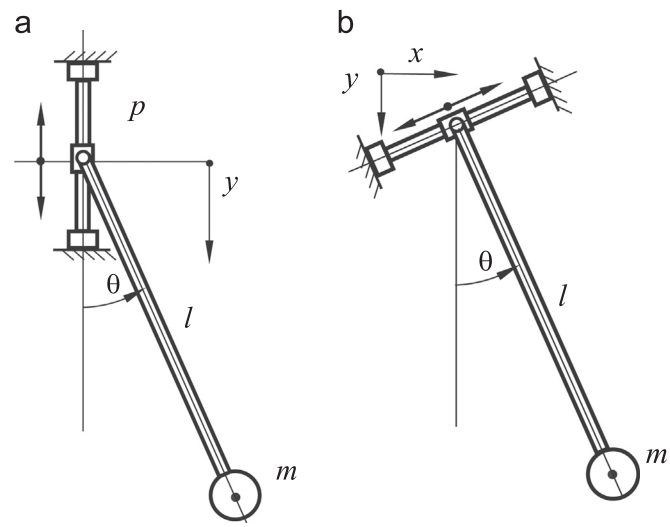

Fig. 1. Schematic representation of (a) vertically excited pendulum and (b) pendulum excited on the plane, adopted from [11].

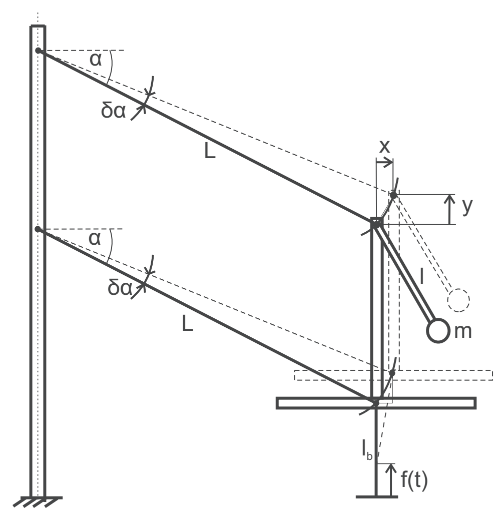

Fig. 2. Schematic representation of the pendulum support geometry and displacements resulting from the vertical forcing $f(t)$.

the (x, y) plane. From the geometry of the set up considered it can be seen that $x=y\tan\alpha$. For small angles between the guide arms and horizontal plane α the resultant vertical displacement y is approximately equal to the wave profile displacement applied at the base $f(t)$. To enable easy comparison between different dynamical systems, for further analysis Eq. (1) has been rewritten in terms of non-dimensional variables and parameters:

$$\theta''+x''\cos\theta+(1+y'')\sin\theta+\gamma\theta'=0, \tag{2}$$

where $''$ and $'$ denote derivatives with respect to the non-dimensional time τ, γ is a damping coefficient, x, y are the base displacements in horizontal and vertical directions and can be calculated from

$$\omega_n=\sqrt{\frac{g}{l}},\quad \tau=\omega_n t,\quad x=\frac{X}{l},\quad y=\frac{Y}{l},\quad \gamma=\frac{c}{m\omega_n}. \tag{3}$$

3. Stochastic wave modelling

In this section the wave excitation function will be derived. The stochastic approach to the wave modelling has been used, taking into account the randomness of the process. The objective of this analysis is to reconstruct a time history of the wave, which could be used for