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A thermal non-equilibrium model with Cattaneo effect for convection in a Brinkman porous layer

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ABSTRACT

This paper aims to investigate the onset of thermal convection in a layer of fluid-saturated Brinkman porous medium taking into account fluid inertia and local thermal non-equilibrium (LTNE) between the solid and fluid phases with Cattaneo effect in the solid. A two-field model is used for the energy equations each representing the solid and fluid phases separately. The usual Fourier heat-transfer law is retained in the fluid phase while the solid phase is allowed to transfer heat via a Cattaneo heat flux theory. It is observed that the Cattaneo effect has a profound influence on the nature of convective instability. In contrast to the standard Brinkman convection with LTNE model, instability is found to occur through oscillatory convection depending on the value of solid thermal relaxation time parameter which in turn depends on other parametric values. The instability characteristics of the system are analyzed in detail for a wide range of parametric values including those for copper oxide and aluminium oxide solid skeletons.

1. Introduction

Buoyancy driven convection in a horizontal porous layer subject to an adverse temperature gradient has been investigated extensively considering local thermal equilibrium (LTE) model. Recently, there has been a great upsurge of interest in understanding this convective instability problem using local thermal non-equilibrium (LTNE) model because of its relevance and applications in many practical situations such as convection in stellar atmospheres, nuclear reactor maintenance, heat exchangers, processing of composite materials, resin flow, fuel cells, tube refrigerators in space, flows in microchannels and porous metallic foams to mention a few [1,2].

In the LTNE model two temperature equations, one for fluid medium and one for solid medium, are used and it appears that the continuum theories for LTNE effects have started in the late 1990s [3–5]. Taking into account LTNE effects, Banu and Rees [6] and Malashetty et al. [7] discussed thermal convection in a porous medium, while Straughan [8] demonstrated the equivalence of linear instability threshold with global non-linear stability threshold for convection. Since then several investigations have been undertaken on the said problem accounting for additional effects such as rotation [8,9], variable viscosity and density maximum [10], non-uniform temperature gradients [11,12], solute concentration [13] and volumetric heat generation [14]. The growing volume of work devoted to this area is well documented in the books by Vafai [15] and Nield and Bejan [16].

The classical energy equation used in the study of convective instability problems in a fluid/porous layer is a parabolic-type partial differential equation which allows an infinite-speed for heat transport. The new theories make use of modified versions that involve hyperbolic-type heat transport equation admitting finite-speed for heat transport. Thus, heat transport is viewed as a wave phenomenon rather than a diffusion phenomenon and this is referred to as second sound. In particular, the second sound effect appears greater in solids, especially those involved in porous metallic foams. A key way to introduce this effect is to use Cattaneo [17] law for the heat flux. Based on this approach, studies have been undertaken in the past to investigate thermal convection in a fluid layer [18,19] and also in a fluid-saturated Darcy porous medium using a local thermal equilibrium (LTE) model with Cattaneo–Fox and Cattaneo–Christov effects [20,21]. The details about the developments on this topic are amply documented in the book by Straughan [22]. The Cattaneo effect on thermal convection in a fluid-saturated Darcy porous medium using a local thermal non-equilibrium (LTNE) model is investigated for the first time by Straughan [2]. In addition to performing linear instability analysis, a global non-linear stability threshold is determined. The effect of second sound is delineated in a detailed manner. A review on thermal instability in a Brinkman porous medium incorporating fluid inertia and Cattaneo–Christov theory in the constitutive equation for heat flux is presented by Haddad [23] for LTE model.

It is known that high porosity porous materials (for example, foam metals) are used in industrial applications such as heat

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exchangers, chemical reactors and fluid filters. Hence, high porosity materials are of much current interest in industry. They are typically man-made and are important in the design of heat transfer devices. In dealing such highly porous materials the use of the higher order Darcy–Brinkman equation becomes more appropriate to model the fluid flow, as opposed to the more commonly used Darcy's law [24]. Some of the works to deal with this extension are by Georgiadis and Catton [25], Kladias and Prasad [26] and Kelliher et al. [27]. Shivakumara et al. [28] investigated the effects of boundary and local thermal non-equilibrium on the criterion for the onset of convection in a sparsely packed horizontal anisotropic porous layer. Many more works are available on the non-Darcy–Benard convection in a porous medium and the details can be found in the books by Straughan [29] and Nield and Bejan [16].

Nonetheless, no attention has been given to understand the influence of Cattaneo effect in the solid on thermal convective instability in high porosity porous materials despite its occurrence and importance in many practical problems as mentioned above. The intent of the present paper is to investigate this problem in a layer of Newtonian fluid-saturated Brinkman porous medium using a LTNE model, which allows the fluid and solid media to be at different temperatures, and including fluid inertia. As considered by Straughan [2], the usual Fourier heat-transfer law is retained in the fluid phase while temperature waves are allowed in the solid phase via a Cattaneo-like heat flux theory as the second sound effect is dominant in the solid skeleton. Although the linear stability analysis is modified, it is still possible to proceed analytically to find the condition for the onset of convection. This work is more general in the sense that the results for the Darcy case can be recovered when the Brinkman or effective viscosity is zero.

2. Mathematical formulation

We consider a horizontal layer of Brinkman porous medium of thickness, d. The lower surface is held at constant temperature, T_l, while the upper surface is at $T_u\ (<T_l)$. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the bottom of the porous layer and the z-axis vertically upward in the presence of gravitational field. The solid and fluid phases of the porous medium are assumed to be in local thermal non-equilibrium (LTNE) with a two-field model for temperatures. The solid temperature equation is modified to allow the heat transfer via a Cattaneo heat flux theory, while the usual Fourier heat-transfer law is used in the fluid. The basic equations governing the flow of an incompressible fluid saturating a layer of Brinkman porous medium with LTNE and Cattaneo effect in the solid are as follows [2,16]:

$$\nabla.\ \vec{q}=0 \tag{1}$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}\right]=-\nabla p+\rho_f\vec{g}-\frac{\mu_f}{K}\vec{q}+\tilde{\mu}_f\nabla^2\vec{q} \tag{2}$$

$$\varepsilon\ (\rho_0\ c)_f\frac{\partial T_f}{\partial t}+(\rho_0\ c)_f\left(\vec{q}\cdot\nabla\right)T_f=\varepsilon\ k_f\nabla^2T_f+h(T_s-T_f) \tag{3}$$

$$(1-\varepsilon)\ (\rho_0\ c)_s\frac{\partial T_s}{\partial t}=-(1-\varepsilon)\ \nabla\cdot\vec{Q}-h(T_s-T_f) \tag{4}$$

$$\tau_s\frac{\partial\vec{Q}}{\partial t}=-\vec{Q}-k_s\nabla T_s \tag{5}$$

$$\rho_f=\rho_0\{1-\alpha_t(T_f-T_l)\} \tag{6}$$

In the above equations, $\vec{q}=(u,v,w)$ is the velocity vector, p the pressure, ρ_f the fluid density, μ_f the fluid viscosity, $\tilde{\mu}_f$ the effective viscosity, K the permeability of the porous medium, ε the porosity of the medium, $\vec{g}$ the gravitational acceleration, T_f the temperature of the fluid, T_s the temperature of the solid, $\vec{Q}$ the heat flux in the solid, c the specific heat at constant pressure, k_f the thermal conductivity of the fluid, k_s the thermal conductivity of the solid, h the inter-phase heat transfer coefficient which depends on the nature of the porous matrix and the saturating fluid, τ_s the solid thermal relaxation time, α_t the coefficient of thermal expansion and ρ_0 the reference density. A subscript s or f refers to the solid or fluid, respectively.

The basic state is quiescent and there exists the following solution for the basic state:

$$\vec{q}_b=0, p_b(z)=p_0-\rho_0 g\,z-\frac{1}{2}\rho_0\alpha_t g\beta\,z^2,$$
$$T_{f\,b}=T_{sb}=\ -\beta z+T_l, \vec{Q}_b=(0,0,k_s\beta) \tag{7}$$

where $\beta=\Delta T/d=(T_l-T_u)/d$ is the temperature gradient and the subscript b denotes the basic state. It may be noted that the fluid and solid phases have the same temperatures at the bounding surfaces of the porous layer.

3. Linear stability theory

To investigate the conditions under which the convection arises against small disturbances, we consider a perturbed state in the following form:

$$\vec{q}=\vec{q}', p=p_b(z)+p', T_f=T_{fb}(z)+T_f', T_s=T_{sb}(z)+T_s', \vec{Q}=\vec{Q}_b(z)+\vec{Q}' \tag{8}$$

where $\vec{q}'=(u',v',w')$, p', T_f', T_s' and $\vec{Q}'=(Q_x',\ Q_y',\ Q_z')$ are the perturbed variables and are assumed to be small. Substituting Eq. (8) into momentum Eq. (2), linearizing, eliminating the pressure term by taking curl twice, the z-component of the resulting equation can be obtained as follows (after dropping the primes):

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}+\frac{\mu_f}{K}-\tilde{\mu}_f\nabla^2\right)\nabla^2 w=\rho_0\alpha_t g\nabla_h^2 T_f \tag{9}$$

where $\nabla_h^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the horizontal Laplacian operator. Eqs. (3) and (4), after using (8) and linearizing, take the following form (after dropping the primes):

$$\varepsilon\ (\rho_0\ c)_f\frac{\partial T_f}{\partial t}+(\rho_0\ c)_f\ w\frac{dT_{fb}}{dz}=\varepsilon\ k_f\nabla^2T_f+h(T_s-T_f) \tag{10}$$

$$(1-\varepsilon)\ (\rho_0\ c)_s\frac{\partial T_s}{\partial t}=-(1-\varepsilon)\ \nabla\cdot\vec{Q}-h(T_s-T_f) \tag{11}$$

Eq. (5), after substituting (8), becomes (after dropping the primes)

$$\tau_s\frac{\partial\vec{Q}}{\partial t}=-\vec{Q}-k_s\nabla T_s \tag{12}$$

It is convenient to eliminate $\vec{Q}$ from Eq. (11), upon using Eq. (12), to get

$$(1-\varepsilon)\ (\rho_0\ c)_s\left(\tau_s\frac{\partial}{\partial t}+1\right)\frac{\partial T_s}{\partial t}=(1-\varepsilon)k_s\nabla^2T_s-h\left(\tau_s\frac{\partial}{\partial t}+1\right)(T_s-T_f) \tag{13}$$

It is seen that Eq. (13) is effectively hyperbolic and oscillatory instability may be possible with increasing τ_s. Eq. (13) becomes parabolic when $\tau_s=0$.

The normal mode expansion of the dependent variables is assumed in the following form:

$$\{w,T_f,T_s\}=\{W(z),\Theta(z),\Phi(z)\}\exp[i(\ell x+my)+\sigma t] \tag{14}$$

where ℓ and m are the wave numbers in the x and y directions, respectively, $W(z)$ is the amplitude of vertical component of perturbed velocity, $\Theta(z)$ is the amplitude of perturbed fluid