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Numerical characterization of effective fully coupled thermo-electro-magneto-viscoelastic-plastic response of smart composites

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ABSTRACT

The focus of the present paper is to construct a general purpose micromechanics model to predict the effective fully coupled time-dependent and non-linear multiphysics responses of smart composites. The present model is established on the basis of the variational asymptotic method and implemented using the finite element method. In light of the time-dependent and non-linear characteristics of composites, an incremental procedure in conjunction with an instantaneous tangential electromagnetomechanical matrix of composites was established. The accuracy of the proposed model was verified through the comparison with ABAQUS results. Finally, a numerical example was employed to demonstrate the capability of the proposed model.

1. Introduction

The smart composite consisting of piezoelectric and piezomagnetic constituents displays a magneto-electric coupling effect that is absent in constituents [1–9]. The magneto-electric coupling effect created by the interaction of piezoelectric phases and piezomagnetic phases has recently been extensively investigated due to their broad engineering applications [10–12]. Since the piezoelectric and piezomagnetic ceramics are brittle and susceptible to fracture, adding a polymer or metallic alloy matrix into the two-phase electromagnetoelastic composite will increase the ductility and formability of the composites. To date, several investigations have been conducted for the response of smart composites containing metallic phases. For example, Bednarcyk [13] developed a micro-macro theory to predict the fully coupled electro-magneto-thermo-elasto-plastic behavior of arbitrary composite laminates using Generalized Method of Cell (GMC). Due to the introduction of linear viscoelastic polymer matrix, the composites exhibit time dependent behavior [14]. The reports that are involved in the response of smart composites containing both metallic phases and viscoelastic phases is still limited. Therefore, there is a need to develop an efficient micromechanical tool for the analysis and design of such composites.

The goal of this paper is to develop a general purpose micromechanics model for predicting the time-dependent, non-linear, and multiphysics response of smart composites. In light of the time-dependent characteristics and non-linearity of constitutive relations, an incremental procedure associated with instantaneous tangential electromechanical matrix was established based on the micromechanics framework VAMUCH [15]. In order to demonstrate the capability, a smart composites consisting of metallic phase, piezoelectric material, piezomagnetic material, and linear viscoelastic matrix was analyzed using the proposed model.

2. Incremental constitutive equations of materials

2.1. Constitutive equations for linear thermo-viscoelastic polymer

Considering the linear thermo-viscoelastic polymer having no history of stress and deformation before time $t=0$, then based on the Boltzmann superposition principle, the constitutive equations for the linear thermo-viscoelastic polymer can be expressed in the time domain in the following way:

$$\sigma_{ij}(t) = \int_0^t \left[B_{ijkl}(t-\tau)\dot{\varepsilon}_{kl}(\tau) + \beta_{ij}(t-\tau)\dot{\theta}(\tau)\right] d\tau \tag{1a}$$

$$D_i(t) = \int_0^t \left[k_{ij}(t-\tau)\dot{E}_j(\tau) + p_i(t-\tau)\dot{\theta}(\tau)\right] d\tau \tag{1b}$$

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$$B_i(t)=\int_0^t\left[\mu_{ij}(t-\tau)\dot{H}_j(\tau)+m_i(t-\tau)\dot{\theta}(\tau)\right]d\tau \tag{1c}$$

where $B_{ijkl}(t)$, $k_{ij}(t)$, and $\mu_{ij}(t)$ are the stress relaxation stiffness, dielectric tensor, and magnetic permeability tensor, respectively; $\dot{\varepsilon}_{kl}(\tau)$ is the strain rate; and $\dot{E}_j(\tau)$ and $\dot{H}_j(\tau)$ are the electric field rate and magnetic field rate, respectively; $\dot{\theta}(\tau)$ is the temperature change rate; $\sigma_{ij}(t)$, $D_i(t)$, and $B_i(t)$ are the instantaneous stress tensor, electrical displacement vector, and magnetic induction vector, respectively; $\beta_{ij}(t)$, $p_i(t)$, and $m_i(t)$ are the instantaneous thermal stress tensor, pyroelectric vector, and pyromagnetic vector, respectively. Note that $\beta_{ij}(t)=-B_{ijkl}(t)\alpha_{kl}$ with α_{kl} being thermal expansion coefficients. In this study, the α_{kl} is assumed to be constant.

According to the time–temperature superposition principle [16], the real time t has to be replaced with reduced time ξ in order to account for the variation of material's properties of polymer with temperature. Hence, Eq. (1a)–(1c) can be rewritten as

$$\sigma_{ij}(t)=\int_0^t\left[B_{ijkl}(\xi-\xi')\dot{\varepsilon}_{kl}(\xi')+\beta_{ij}(\xi-\xi')\dot{\theta}(\xi')\right]d\xi' \tag{2a}$$

$$D_i(t)=\int_0^t\left[k_{ij}(\xi-\xi')\dot{E}_j(\xi')+p_i(\xi-\xi')\dot{\theta}(\xi')\right]d\xi' \tag{2b}$$

$$B_i(t)=\int_0^t\left[\mu_{ij}(\xi-\xi')\dot{H}_j(\xi')+m_i(\xi-\xi')\dot{\theta}(\xi')\right]d\xi' \tag{2c}$$

The reduced time $\xi=\xi(t)$ is defined by

$$\xi(t)=\int_0^t\frac{dt'}{a_T} \tag{3}$$

where a_T is a time-scale shift factor, and $\xi'=\xi(\tau)$.

As pointed out by Pyatigorets et al. [17], since the corresponding value of real time t can be found for each value of reduced time ξ and vice versa, the stress and strain in the reduced time domain can be replaced with their values found for the corresponding real time, such that

$$\sigma_{ij}(\xi)\equiv\sigma_{ij}(\xi(t))\equiv\sigma_{ij}(t),\quad \varepsilon_{ij}(\xi)\equiv\varepsilon_{ij}(\xi(t))\equiv\varepsilon_{ij}(t) \tag{4}$$

Hence, the Eq. (2a)–(2c) can be simplified as

$$\sigma_{ij}(t)=\int_0^t\left[B_{ijkl}(\xi(t)-\xi(\tau))\dot{\varepsilon}_{kl}(\tau)+\beta_{ij}(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau \tag{5a}$$

$$D_i(t)=\int_0^t\left[k_{ij}(\xi(t)-\xi(\tau))\dot{E}_j(\tau)+p_i(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau \tag{5b}$$

$$B_i(t)=\int_0^t\left[\mu_{ij}(\xi(t)-\xi(\tau))\dot{H}_j(\tau)+m_i(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau \tag{5c}$$

In light of the non-linear, time dependent, and multiphysics response of the composites, our analysis need to be incremental. The incremental formulations of Eq. (5a)–(5c) can be expressed as

$$\begin{aligned}\Delta\sigma_{ij}(t)&=\sigma_{ij}(t+\Delta t)-\sigma_{ij}(t)\\&=\int_t^{t+\Delta t}\left[B_{ijkl}(\xi(t+\Delta t)-\xi(\tau))\dot{\varepsilon}_{kl}(\tau)+\beta_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad+\int_0^t\left[B_{ijkl}(\xi(t+\Delta t)-\xi(\tau))\dot{\varepsilon}_{kl}(\tau)+\beta_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad-\int_0^t\left[B_{ijkl}(\xi(t)-\xi(\tau))\dot{\varepsilon}_{kl}(\tau)+\beta_{ij}(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\end{aligned} \tag{6a}$$

$$\begin{aligned}\Delta D_i(t)&=D_i(t+\Delta t)-D_i(t)\\&=\int_t^{t+\Delta t}\left[k_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{E}_j(\tau)+p_i(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad+\int_0^t\left[k_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{E}_j(\tau)+p_i(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad-\int_0^t\left[k_{ij}(\xi(t)-\xi(\tau))\dot{E}_j(\tau)+p_i(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\end{aligned} \tag{6b}$$

$$\begin{aligned}\Delta B_i(t)&=B_i(t+\Delta t)-B_i(t)\\&=\int_t^{t+\Delta t}\left[\mu_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{H}_j(\tau)+m_i(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad-\int_0^t\left[\mu_{ij}(\xi(t+\Delta t)-\xi(\tau))\dot{H}_j(\tau)+m_i(\xi(t+\Delta t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\\&\quad-\int_0^t\left[\mu_{ij}(\xi(t)-\xi(\tau))\dot{H}_j(\tau)+m_i(\xi(t)-\xi(\tau))\dot{\theta}(\tau)\right]d\tau\end{aligned} \tag{6c}$$

Although the strain rate, electrical field rate, and magnetic field rate are not necessarily constant in the whole time domain, it is reasonable to assume that the strain rate, electrical field rate, and magnetic field rate are kept constant during each time increment Δt. The temperature change rate can be kept uniform in the whole composites. Then, the Eq. (6a)–(6c) can be rephrased as

$$\Delta\sigma_{ij}(t)=L_{ijkl}(t)\Delta\varepsilon_{kl}(t)+\gamma_{ij}(t)\Delta\theta(t)+\omega_{ij}(t) \tag{7a}$$

with

$$\begin{aligned}L_{ijkl}(t)&=\frac{1}{\Delta t}\int_t^{t+\Delta t}B_{ijkl}[\xi(t+\Delta t)-\xi(\tau)]d\tau\\\gamma_{ij}(t)&=\frac{1}{\Delta t}\left(\int_t^{t+\Delta t}\beta_{ij}[\xi(t+\Delta t)-\xi(\tau)]d\tau\right)\\\omega_{ij}(t)&=\int_0^t\left[B_{ijkl}(\xi(t+\Delta t)-\xi(\tau))-B_{ijkl}(\xi(t)-\xi(\tau))\right]\dot{\varepsilon}_{kl}(\tau)\,d\tau\\&\quad+\int_0^t\left[\beta_{ij}(\xi(t+\Delta t)-\xi(\tau))-\beta_{ij}(\xi(t)-\xi(\tau))\right]\dot{\theta}(\tau)d\tau\end{aligned}$$

$$-\Delta D_i(t)=-K_{ik}(t)\Delta E_k(t)-P_i(t)\Delta\theta-\varpi_i(t) \tag{7b}$$

with

$$\begin{aligned}K_{ik}(t)&=\frac{1}{\Delta t}\int_t^{t+\Delta t}k_{ik}(\xi(t+\Delta t)-\xi(\tau))d\tau\\P_i(t)&=\frac{1}{\Delta t}\left(\int_t^{t+\Delta t}p_i(\xi(t+\Delta t)-\xi(\tau))d\tau\right)\\\varpi_i(t)&=\int_0^t\left[k_{ij}(\xi(t+\Delta t)-\xi(\tau))-k_{ij}(\xi(t)-\xi(\tau))\right]\dot{E}_j(\tau)\,d\tau\\&\quad+\int_0^t\left[p_i(\xi(t+\Delta t)-\xi(\tau))-p_i(\xi(t)-\xi(\tau))\right]\dot{\theta}(\tau)d\tau\end{aligned}$$

$$-\Delta B_i(t)=-N_{ik}(t)\Delta H_k(t)-M_i(t)\Delta\theta-\Psi_i(t) \tag{7c}$$

with

$$\begin{aligned}N_{ik}(t)&=\frac{1}{\Delta t}\int_t^{t+\Delta t}\mu_{ik}(\xi(t+\Delta t)-\xi(\tau))d\tau\\M_i(t)&=\frac{1}{\Delta t}\left(\int_t^{t+\Delta t}m_i(\xi(t+\Delta t)-\xi(\tau))d\tau\right)\\\Psi_i(t)&=\int_0^t\left[\mu_{ij}(\xi(t+\Delta t)-\xi(\tau))-\mu_{ij}(\xi(t)-\xi(\tau))\right]\dot{H}_j(\tau)\,d\tau\\&\quad+\int_0^t[m_i(\xi(t+\Delta t)-\xi(\tau))-m_i(\xi(t)-\xi(\tau))]\dot{\theta}(\tau)d\tau\end{aligned}$$

2.2. Constitutive equations for piezoelectric-piezomagnetic materials

The elastic and the dielectric responses are coupled in piezoelectric materials where the mechanical variables of stress, and strain are related to each other as well as to the electric variables of electric field and electric displacement. The coupling between mechanical and electric fields is described by piezoelectric coefficients. The linear rate independent coupled constitutive equations of piezoelectric materials are given by

$$\sigma_{ij}=C^e_{ijkl}\varepsilon_{kl}-e_{ijk}E_k-q_{ijk}H_k+\beta_{ij}\theta \tag{8a}$$

$$D_i=e_{ikl}\varepsilon_{kl}+k_{ik}E_k+a_{ik}H_k+p_i\theta \tag{8b}$$