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Dynamic buckling of partially-sway frames with varying stiffness using catastrophe theory

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ABSTRACT

This work deals with the static and dynamic stability analysis of imperfect partially-sway frames with non-uniform columns. The examined two-bar frames are elastically supported and subjected to an eccentrically vertical load at their joint. Through a linear stability analysis, the effect of the taper ratio of the column cross-section on the buckling capacity of the partially-sway frame is thoroughly discussed. Using a non-linear method an accurate formula has been established for determining the exact asymmetric bifurcation point associated with the maximum load carrying capacity. These findings have been re-derived more readily using Catastrophe Theory (CT) and considering the frame as a one degree-of-freedom (1-DOF) system through an efficient technique. A local analysis allows us to classify, after reduction, the total potential energy (TPE) function of the system to one of the seven elementary Thom's catastrophes (with known properties) and to obtain static and dynamic singularity as well as bifurcational sets. It has been found that geometrical and loading imperfections, which are always present in structural engineering problems, have a significant effect on the dynamic buckling loads. The efficiency of the present approach is illustrated via several examples, while results from finite element analyses are in good agreement with the analytical solution presented herein.

1. Introduction

In recent years instability phenomena occurring in steel structures are of paramount importance to engineers since they are fully associated with their maximum strength. Towards this direction, a unified view has been established in conjunction with Bifurcation and Catastrophe Theory [1], which are successfully applied to structural stability, particularly in view of their recent progress [2–5]. Indeed, Catastrophe Theory (CT) is widely accepted as providing a powerful universal method for the study of discontinuities, singularities and instabilities in the behavior of systems. Therefore, solutions of general validity (universal solutions) are derived by classifying the system's total potential energy function into one of the seven Thom's elementary catastrophes [6–9], while one can also deal with non-linear dynamic stability problems using CT [10].

The problem of post-buckling behavior of imperfect two-bar frames using CT has been examined by several researchers, such as Kounadis [9] and Raftoyiannis et al. [11], while Ioannidis et al. [12,13] has studied the effect of imperfections associated with the load-carrying capacity of a sway or a non-sway frame through bifurcational instability. More recent studies have contributed to the investigation of the influences of initial geometric imperfections on the stability of structural systems via energy approach, by Vakakis et al. [14,15] and Orlando et al. [16,17].

In addition, a number of researchers have dealt with steel frames with non-prismatic sections [18–20]. However, the aforementioned techniques have not been applied to such structural systems. Instead, these frames have been analyzed via linear or linearized methods unable to capture their postbuckling behavior and the significant reduction of the load-carrying capacity due to the existence of unavoidable imperfections. This work aims at presenting a qualitative stability analysis of partially-sway frames with compression members (columns) of varying both cross-sectional area and moment of inertia. The examined two-bar frame is loaded eccentrically by a concentrated vertical load on its joint. The column is elastically restrained and the girder is hinged on a horizontally movable support with elastic spring. The analysis of the problem is based on non-linear kinematic relations and moderately large rotations with small axial strains, taking into account variations of stiffness and cross-sectional area of the I-column. This frame has been studied using both linear and non-linear approaches [20–22]. Moreover, using moderate stability theory and non-linear stability analysis it is proven that the load carrying capacity of such frames is associated with asymmetric bifurcation in their ideal state.

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Subsequently, Catastrophe Theory is properly employed by considering the continuous system (frame) as one degree-of-freedom (1-DOF) with generalized coordinate the axial force, main control parameter the loading and normal control parameter the imperfection. Through a local analysis via Taylor's expansion of the non-linear equilibrium equation we can classify the total potential energy (TPE) function of the system into one of the 7 canonical forms of the corresponding universal unfolding with a-priori known properties. Thereafter, using energy criteria the static catastrophe is extended to the corresponding dynamic catastrophe of undamped frames under step loading. Attention is focused on the determination of the static and dynamic bifurcational sets, as well as on the comparison of the static singularity set with the respective exact critical loads. The effect of geometrical and loading imperfections on the dynamic buckling load of such frames is thoroughly investigated. Moreover, the effect of imperfections, which reduces the system into a limit point associated with significant reduction of the load carrying capacity, is properly established. Finally, the reliability of the computational work is fully discussed and a comparison with numerical results is presented in both graphical and tabular form.

2. Linear bifurcational buckling analysis

The following analysis concerns plane frames with tapered columns of varying cross-section (varying moment of inertia and cross-sectional area). A steel I-column with linearly varying web and flanges along its height is shown in Fig. 1.

The cross-sectional area A_1 and the moment of inertia I_1 at an arbitrary point x_1 are given by

$$A_1(\bar{x}_1)=A_0\frac{\bar{x}_1}{\bar{\alpha}}=A_m\frac{2\bar{x}_1}{2\bar{\alpha}+l_1},$$
$$I_1(\bar{x}_1)=I_0\left(\frac{\bar{x}_1}{\bar{\alpha}}\right)^3=I_m\left(\frac{2\bar{x}_1}{2\bar{\alpha}+l_1}\right)^3, \tag{1}$$

where $\bar{x}_1$ (see also Fig. 2) is the coordinate along the length of the column, and A_0, I_0 and A_m, I_m are the cross-sectional area and moment of inertia at the bottom and mid-length of the column, respectively, computed from the flanges and web width linear variation (from b_{f1} and h_1 at the bottom to b_{f2} and h_2 at the top) while t_f and t_w is the thickness of the flange and web, respectively.

In Fig. 2, the mathematical model used for linear buckling analysis is shown. The horizontal member is assumed to be prismatic with constant area A_2 and constant moment of inertia I_2 along its length. Furthermore, the frame is supported at A by a pinned support with a rotational spring of stiffness $\bar{c}_r$, and at C by a roller support with a translational spring of stiffness $\bar{c}_t$, which simulates the horizontal stiffness of the frame.

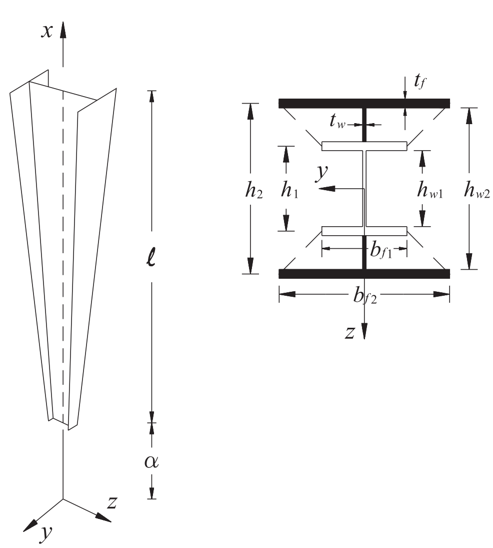

Fig. 1. Steel I-column with linearly varying cross-sectional dimensions.

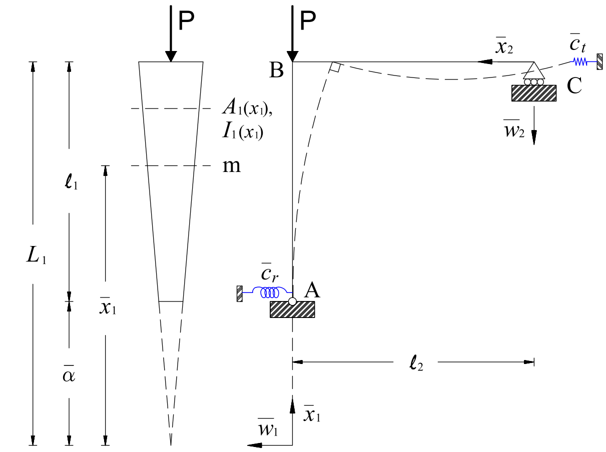

Fig. 2. Geometry and sign convention of a perfect rectangular frame with varying cross-section.

If $\bar{w}_1(x_1)$ and $\bar{w}_2(x_2)$ are the transverse deflections of the column and the girder, respectively, the governing differential equations for linear buckling are

$$\left[EI_m\cdot\left(\frac{2\bar{x}_1}{l_1+2\bar{\alpha}}\right)^3\cdot\bar{w}''_1(\bar{x}_1)\right]''+P\cdot\bar{w}''_1(\bar{x}_1)=0,$$
$$EI_2\bar{w}_2''''(\bar{x}_2)=0, \tag{2}$$

Introducing the non-dimensional quantities $x_i=\bar{x}_i/l_i$, $w_i=\bar{w}_i/l_i$ (for $i=1,2$)

$$\alpha=\bar{\alpha}/l_1,\quad \mu^2=\beta^2\left(\frac{1}{2}+\alpha\right)^3,\quad \beta^2=\frac{Pl_1^2}{EI_m},\quad r=\frac{l_2}{l_1},\quad \rho=\frac{I_m}{I_2},\quad \alpha=\frac{1}{l_1'}=\frac{\bar{\alpha}}{l_1}$$

and also for the rotational and the translational springs

$$c_r=\frac{\bar{c}_r l_1}{EI_0},\quad c_t=\frac{\bar{c}_t l_2^3}{EI_2}$$

the corresponding boundary conditions are

$$w_1(\alpha)=w_2(0)=w_2(1)=0,\quad w_1'(1+\alpha)=w_2'(1),$$
$$w_1''(\alpha)-c_r w_1'(\alpha)=0,\quad w_2''(0)=0,$$
$$-\left[\left(\frac{2x_1}{2\alpha+1}\right)^3 w_1''x_1)\right]'_{x_1=1+\alpha}-\beta^2 w_1'(1+\alpha)=-c_t w_1(1+\alpha)\frac{1}{\rho r^3},$$
$$\left(\frac{2+2\alpha}{1+2\alpha}\right)^3 w''_1(1+\alpha)+\frac{1}{\rho r}w''_2(1)=0. \tag{3}$$

The general solutions of Eq. (2) in dimensionless form are

$$w_1(x_1)=C_1\sqrt{x_1}BesselJ\left(1,\frac{2\mu}{\sqrt{x_1}}\right)+C_2\sqrt{x_1}BesselY\left(1,\frac{2\mu}{\sqrt{x_1}}\right)+C_3x_1+C_4,$$
$$w_2(x_2)=D_1x_2^3+D_2x_2^2+D_3x_2+D_4 \tag{4}$$

The above expressions have been obtained by integrating Eq. (2). The first one is a 4th order differential equation with cubic order coefficient where coordinate transformation has been applied on x_1, while the second one is a classical 4th order differential equation with constant coefficient. For easier supervision of the equations we use the following abbreviations

$$J_{a,1}=BesselJ\left(1,\frac{2\mu}{\sqrt{\alpha}}\right),Y_{a,1}=BesselY\left(1,\frac{2\mu}{\sqrt{\alpha}}\right),$$
$$J_{a,0}=BesselJ\left(0,\frac{2\mu}{\sqrt{\alpha}}\right),\ Y_{a,0}=BesselY\left(0,\frac{2\mu}{\sqrt{\alpha}}\right)$$