International Journal of Non-Linear Mechanics 71 (2015) 127–131

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

A remark on similarity analysis of boundary layer equations of a class of non-Newtonian fluids

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ARTICLE INFO

Article history:
Received 20 August 2014
Received in revised form
28 October 2014
Accepted 29 October 2014
Available online 6 November 2014

Keywords:
Non-Newtonian fluids
Power-law fluid
Implicit fluid theory
Reiner–Philippoff fluids
Similarity solutions

ABSTRACT

A similarity analysis of three-dimensional boundary layer equations of a class of non-Newtonian fluid in which the stress, an arbitrary function of rates of strain, is studied. It is shown that under any group of transformation, for an arbitrary stress function, not all non-Newtonian fluids possess a similarity solution for the flow past a wedge inclined at arbitrary angle except Ostwald-de-Waele power-law fluid. Further it is observed, for non-Newtonian fluids of any model only 90° of wedge flow leads to similarity solutions. Our results contain a correction to some flaws in Pakdemirli's [14] (1994) paper on similarity analysis of boundary layer equations of a class of non-Newtonian fluids.

1. Introduction

The classical theory of Newtonian fluid depends upon the hypothesis of linear relationship between stress tensor and strain tensor, rate of strain tensor and even rate of stress tensor. The fluids which do not follow such a linear relationship are called non-Newtonian fluid. The non-Newtonian fluids are usually classified as follows: fluids in which shear stress depends on the rates of shear only; fluids for which relation between shear stress and rates of strain depends on time and the viscoinelastic fluids which possess both elastic and viscous properties. Thus the mathematical structure of the shearing stress and the rate of shear plays a vital role in describing any non-Newtonian fluid. It is quite difficult to provide a single constitutive relation that can be used to describe a non-Newtonian fluid due to a great diversity found in its physical structure. That is why for many non-Newtonian fluid models this relation may be empirical or semi-empirical.

Constitutive expression of the stress within the context of classical continuum mechanics provides explicit relationships for the stress in terms of appropriate kinematical quantities and the density. In contrast many constitutive relations for inelastic and viscoelastic fluids are implicit relations. Rajagopal [1] has discussed a more generalized implicit constitutive relation for an incompressible fluid wherein the viscosity depending on the pressure. His proposed framework allows researchers to construct models for turbulent flows. In an another recent paper on implicit fluid theories, Saccomandi and Vergori [2] have studied in detail the flow down an inclined plane of such fluid by using the well established theory of lubrication that has been developed by Rajagopal and Szeri [3]. They considered flow regimes, namely flows wherein viscous effects, surface tension effects, etc., are predominant. In quasi-steady flow they showed that, the breaking time of the waves is delayed in comparison to the classical Newtonian fluid. In the viscous regimes, they found that if the fluid viscosity is affected by the pressure changes, then the traveling waves could be both qualitatively and quantitatively different from those occurring in a fluid with constant viscosity. However their work lacks an accurate investigation of other non-Newtonian effects on the fluid flow that has been studied by Rajagopal and co-workers [4]. They carry out an analysis of the flow of fluid with pressure and shear rate dependent viscosity down an inclined plane within the context of the lubrication approximation.

Among all the non-Newtonian fluid models, the Ostwald-de-Waele power-law model is most popular and versatile for the simplicity of its stress strain relationship. It describes a great number of real non-Newtonian fluids. The use of this model assumes fluid to

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http://dx.doi.org/10.1016/j.ijnonlinmec.2014.10.022
0020-7462/

be purely viscous. There are certain flaws of this model. First the stress constitutive relation is derived from an empirical relation and hence dimension of one parameter depends upon that of other. Second it reveals an infinite effective viscosity for low shear rate, that limiting its range of applicability. Although other non-Newtonian models like Prandtl–Eyring, Powel–Eyring and Williamson, etc. are mathematically more complex, attracts our attention as their stress constitutive relation can be derived using the kinetic theory of liquids rather than an empirical relation. Also these models correctly reduce to Newtonian behavior for low as well as high shear rates.

The formulation of rheological relations between deviatoric stress component τ_{ij} and the strain rate component e_{ij} for different non-Newtonian fluids and their evaluation in terms of known variables under the proper boundary layer assumption is indeed a quite difficult task. Schowalter [5] was the first to derive such relationship for the three-dimensional incompressible boundary layer equations of non-Newtonian power-law fluids. He has shown that the similarity solution of such equations exists only if free stream velocities in X-direction are constant. For the case of two dimensional jet flow the same model was discussed by Kapur [6]. Later on, Na and Hansen [7] have extended Schowalter [5] analysis with the more general conclusion that for the similarity solutions, the free stream velocities in the X-direction must differ by a multiplicative constant. In the literature it has been found that, the similarity solutions of boundary layer equations of non-Newtonian fluids past static surfaces were also examined by Acrivos et al. [8], Wells [9], Hayasi [10], Lee and Ames [11], Hansen and Na [12], Timol and Kalthia [13] and Pakdemirli [14].

The work under investigation considers the two papers published in the past two decades one by Pakdemirli [14] and the other by Na [15]. It is surprising to note that what is proved by Pakdemirli [14] is again well established by Na [15], without verifying the result obtained by Pakdemirli [14]. We are against the opinion that Pakdemirli [14] has wrongly interpreted the conclusion drawn by Timol and Kalthia [13] and that of Hansen and Na [12]. The author [14] has investigated the three-dimensional incompressible boundary-layer flow of a class of non-Newtonian fluids where the shear stress in boundary-layer is an arbitrary function of the rate of strain. He obtained similarity solution under a spiral group of transformation for the class of non-Newtonian fluids past a wedge inclined at an arbitrary angle. Moreover claims that the similarity solutions he obtained are independent of the use of stream function and are far better than others [12,13]. The present work discusses some flaws in Pakdemirli's [14] paper on similarity analysis of three dimensional boundary layer equations of a class of non-Newtonian fluids. It is shown that the author [14] has misinterpreted the conclusion drawn by Timol and Kalthia [13] and consequently reported some incorrect solutions. The present analysis introduces a method of formulation and solution which can be applied to the three-dimensional boundary layer flow of any non-Newtonian fluid over any body shape in which the velocity gradient is expressed explicitly as a function of the shearing stress.

2. Problem formulation

In the literature it is found that Lee and Ames [11] are probably the first who have considered stress–strain relationship for non-Newtonian fluids in the form of arbitrary functions as Eq. (1) and investigated similarity solutions for different flow geometries of non-Newtonian fluids.

$$\tau_{xy} = G(\partial u/\partial y) \tag{1}$$

Following Lee and Ames [11], Hansen and Na [12] have proposed the more general stress–strain relationship as,

$$F(\tau_{xy}; \partial u/\partial y) \tag{2}$$

Further they have shown that similarity solutions for all non-Newtonian fluids characterized by Eq. (2) exist for flow past 90° wedges only. They listed up six non-Newtonian fluid models characterized by Eq. (2). The mathematical formulation of functional relationship between shearing stress and rate of strain of viscoinelastic non-Newtonian fluids in the form of tensor notations was probably first suggested by Timol and Kalthia [13]. This functional relationship may be of three types:

1. Shearing stress may be an explicit function of the rate of strain. For example: Newtonian fluids, non-Newtonian power-law fluids and some second and third order fluids.
2. Shearing stress may be an implicit function of the rate of strain. For example: Reiner–Philipoff fluids, Ellis fluids, Eyring viscous fluids [7].
3. Shearing stress may be a composite function of the rate of strain.
   For example: Powell–Eyring fluids, Prandtl–Eyring fluids, Sisco fluids, Sutterby fluids, Williamson fluids, etc. [7].

Mathematically, both the composite and implicit types of stress–strain relationships are given by Eq. (2). Timol and Kalthia [13] have extended the results obtained by Hansen and Na [12] for the three-dimensional boundary layer flow of non-Newtonian fluids. Further they have clearly stated that similarity solutions for all non-Newtonian fluids characterized by the property that its deviatoric stress tensor τ_{ij} related to rate of deformation tensor e_{ij} by an arbitrary continuous function as,

$$F(\tau_{ij}; e_{ij}) = 0 \tag{3}$$

exist only for the flow past a 90° wedge (note that Eqs. (2) and (3) are equivalent). It is a well-known fact that for non-Newtonian fluids characterized by Eq. (1) similarity solution exists for the flow past a wedge inclined at any arbitrary angle. Pakdemirli [14] has considered an arbitrary stress function which depends on the rate of strain (for the three-dimensional boundary layer) in the following form:

$$\tau_{xy} = \tau_{xy}(\partial u/\partial y, \partial w/\partial y) \tag{4}$$

$$\tau_{yz} = \tau_{yz}(\partial u/\partial y, \partial w/\partial y) \tag{5}$$

Eqs. (4) and (5) are identically the same as Eq. (1). In other words, Pakdemirli [14] has made the same assumption that was made long back by Lee and Ames [11]. Further, he has stated that Prandtl, Powell–Eyring, Williamson, power-law type of fluids are examples of functions (4) and (5). This statement is also incorrect, because except power-law fluids no other fluid bears stress–strain relationship given by Eqs. (4) and (5). Here it is worth to note that though the author [14] has cited Hansen and Na [12], it seems that he has not thoroughly studied the stress strain relationship for the different non-Newtonian fluid models. Thus conclusions made by Pakdemirli [14] in his paper are all incorrect. In the next section, similarity analysis is made of three-dimensional boundary flow of a Reiner–Philippoff non-Newtonian fluid which will focus on the flaws reported by Pakdemirli [14].

3. Similarity analysis

Consider the governing differential equations for the three-dimensional boundary layer flow of Reiner–Philippoff non-Newtonian fluid given by Na and Hansen [7] and Timol and Kalthia [13].

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$