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A series solution for the in-plane vibration analysis of orthotropic rectangular plates with elastically restrained edges

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ABSTRACT

In this paper, a series solution for the free in-plane vibration analysis of orthotropic rectangular plate with elastically restrained edges is obtained using an two-dimensional (2-D) improved Fourier series method. Both two in-plane displacements are represented by a double Fourier cosine series and four supplementary functions, in the form of the product of a polynomial function and a single cosine series expansion, introduced to remove the potential discontinuities associated with the original displacement functions along the edges when they are viewed as periodic functions defined over the entire x–y plane. All the unknown expansion coefficients are sought in a strong form by letting the solution accurately satisfy both the boundary conditions and the governing differential equations on a point-wise basis. Numerical examples are presented to demonstrate the reliability and effectiveness of the current solution through the comparison with those obtained from other analytical approach as well as Finite Element Analysis (FEA) by using NASTRAN.

1. Introduction

Rectangular plate is extensively used as basic structural element in various engineering braches, such as aerospace, building and marine engineering. A clear understanding on its dynamic characteristics is necessary to ensure the reliable design of such type structures. For this reason, a great amount of research effort has been devoted to the study of vibration analysis of plate structure for many years [1,2]. However, the majority of these investigations are mainly restricted to the transverse vibration, and relatively little attention has been paid to its in-plane counterpart. Recently, some investigations have shown that the in-plane vibration plays important role in the energy transmission between coupled plates [3–5], modeling of sandwich composite plate [6], as well as the development of ultrasonic motor [7], and so on. For the in-plane vibration problem, it is widely known that its resonant frequencies are much higher than those of its transverse counterpart due to the larger longitudinal stiffness. For the calculation by using FEA, it is needed to use much more mesh to capture its modal behavior, which means that the more computational resource is required. Another issue related with FEA is that the model should be rebuilt, and remeshing is also required for any variation of geometrical parameters, such as the aspect ratio, is changed. Then, the in-plane vibration analysis of rectangular plates by analytical approaches has recalled the scientific interest of many researchers in the past decade.

Bardell et al. [8] have made a significant contribution to the community of in-plane vibration. They not only presented a relatively complete reference list of research work performed in the earlier stage but also several results have also been obtained for the first time, which can be used as the validation benchmark for the subsequent development of other modeling approaches. Farag and Pan [9] developed a solution for the free vibrational characteristics of rectangular plates with two parallel edges clamped, in which they examined the propagation and attenuation frequency bands and cutoff frequencies, and expressions were derived for the natural frequencies of coupled and uncoupled modes. Gorman [10] introduced the superposition method, formerly developed for the lateral plate vibration problems [11], as a means for obtaining analytical-type solutions for free in-plane vibration of rectangular plates with completely free boundary restraints. Making use of such superposition method, he subsequently derived accurate analytical type solutions for the free in-plane vibration frequencies and mode shapes of clamped and simply supported rectangular plates [12]. By using the direct separation of variables, Xing and Liu [13] found the exact solutions of in-plane natural frequencies and mode shapes of rectangular plates with at least two opposite edges simply supported.

Beside the aforementioned studies on classical boundary conditions (namely free, simply supported and clamped), in-plane vibration problem of rectangular plate with complicated edge

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Nomenclature

a	length of a plate
b	width of a plate
E_x	Young's modulus related to the x direction
E_y	Young's modulus related to the y direction
E_x/E_y	stiffness ratio
G_{xy}	shear stiffness
k	stiffness of the restraining springs
$\overline{K}$	non-dimensional stiffness ($\overline{K}=ka(1-\mu_x\mu_y)/\sqrt{E_xE_y}$)
$u(x, y)$	in-plane displacement component in the x-direction
$v(x, y)$	in-plane displacement component in the y-direction
δ_{mn}	Kronecher delta function
λ_{am}	$\lambda_{am}=m\pi/a$
λ_{bn}	$\lambda_{bn}=n\pi/b$
μ_x	Poisson's ratio related to the x-direction
μ_y	Poisson's ratio related to the y-direction
σ_x, σ_y	in-plane normal stresses
τ_{xy}	in-plane shear stress
ω	angular frequency
γ	dimensionless frequency ($\gamma=\omega a\sqrt{\rho/G_{xy}}/\pi$)

supports are also considered by several researchers. Gorman [14] employed the superposition method to carry out the free in-plane vibration analysis of rectangular plates with elastic support normal to the boundaries, while the plate has the classical boundary conditions along the tangential direction on each edge. Du et al. [15] proposed a 2D (two dimensional) improved Fourier series method for the in-plane vibration analysis of plate structure with elastically restrained edges both along normal and tangential directions, in which the two in-plane displacement components are constructed as the combination of 2D standard Fourier series and the products of polynomial and single Fourier cosines series with the aim to remove all the potential discontinuities encountered on each edge. Subsequently, this solution framework was employed to predict the free in-plane vibration characteristics of rectangular plates with arbitrary elastically point-supported edges [16]. Moreover, Dozio [17] developed a Ritz method using a set of trigonometric functions to obtain accurate in-plane modal properties of rectangular plates with non-uniform elastic edge restraints.

A review of the scientific literature in this field reveals that the majority of existing in-plane vibration investigation is mainly devoted to the isotropic plate which has the same material property along different directions, while the reported work on the in-plane vibration of orthotropic plate is little. Gorman [18] applied the superposition method for the in-plane vibration analysis of clamped orthotropic plates. Liu and Xing [19] presented the exact solutions for the free in-plane vibrations of orthotropic rectangular plates by the separation of variable method, plenty of exact eigensolutions for free in-plane vibrations of orthotropic rectangular plates with at least two simply supported opposite edges are derived. Dozio [20] developed accurate solutions for the free in-plane vibrations of single-layer and symmetrically laminated rectangular composite plates with an arbitrary combination of clamped and free boundary conditions by the Ritz method with a set of trigonometric functions. It can be found that the literature on the in-plane vibration of orthotropic rectangular plate is confined to the cases when classical boundary conditions are applied.

Motivated by the limitation of boundary conditions in the current studies of the in-plane vibration analysis of orthotropic plate structures, in this paper, an exact series solution for the free in-plane vibration analysis of orthotropic rectangular plate with elastically restrained edges is developed by using 2-D improved Fourier series method, previously proposed by Du et al.[15] for the in-plane vibration analysis of isotropic plate structure. The two in-plane displacement field functions are both expressed as the superposition of a 2-D Fourier cosine series and four supplementary functions in the form of the product of a polynomial function and a single cosine series expansion, with all these unknown coefficients solved from the governing differential equations and the boundary conditions in the strong form. Sufficient details are described on the solving procedure. Then, the accuracy and effectiveness of the current solution are validated through the comparison with other approaches in literature or the finite element method. Finally, some concluding remarks are made.

2. Theoretical formulations

2.1. Elastically restrained boundary constraint equations

Consider an orthotropic rectangular plate, with the dimension of $a\times b$, and the coordinate system used in this study is illustrated in Fig. 1. Two sets of restraining springs are assumed to be distributed uniformly along each edge, then all the classical boundary conditions as well as their combinations can be easily obtained by simply setting the spring coefficients into zero or infinity. For example, the clamped boundary condition can be readily obtained by setting the spring coefficients into infinity (a very large number in practical calculation) for both the normal and tangential restraining springs on each edge.

For the orthotropic rectangular plate, which has different material properties in different directions, making use of the strain–stress relationship defined in elasticity theory, the in-plane normal and shear stresses can be expressed as follows:

$$\sigma_x=A_{11}\frac{\partial u}{\partial x}+A_{12}\frac{\partial v}{\partial y}, \tag{1}$$

$$\sigma_y=A_{21}\frac{\partial u}{\partial x}+A_{22}\frac{\partial v}{\partial y}, \tag{2}$$

and

$$\tau_{xy}=G_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{3}$$

where $A_{11}=E_x/(1-\mu_x\mu_y)$, $A_{12}=\mu_xE_x/(1-\mu_x\mu_y)$, $A_{21}=\mu_yE_y/(1-\mu_x\mu_y)$ and $A_{22}=E_y/(1-\mu_x\mu_y)$ are the stretch stiffness, in view of the Betti principle, $A_{12}=A_{21}$; the u and v are the in-plane displacements in

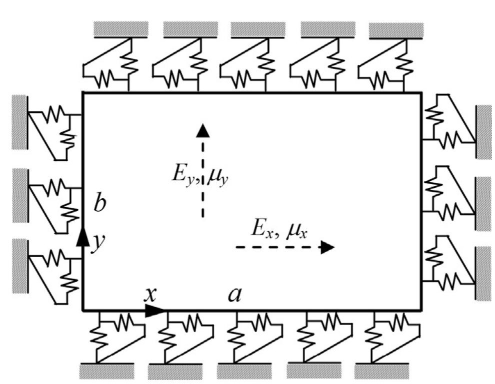

Fig. 1. An orthotropic rectangular plate with elastically restrained edges.