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Size-dependent bending analysis of Kirchhoff nano-plates based on a modified couple-stress theory including surface effects

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ABSTRACT

In the present work, a new Kirchhoff plate model is developed using a modified couple-stress theory to study the bending behavior of nano-sized plates, including surface energy and microstructure effects. The surface elasticity theory of Gurtin and Murdoch is used to model the surface energy effects, into the framework of the modified couple-stress theory of elasticity. Newtonian continuum mechanics approach is used to derive the differential form of the equilibrium equations for the modified Kirchhoff plate theory.

The modified plate rigidity is derived to express the size effects in nanoplates. Presence of a length scale parameter, in the context of the modified couple-stress theory, enables us to express the size effect in nano-scale structures. In addition, an intrinsic length scale parameter is determined as a result of taking surface energy effects into account.

In order to illustrate the model, an analytical solution of the static bending of a simply supported nano-plate has been derived. For ultra-thin plates it is noticed that the microstructure effects on bending rigidity and deflection, through the application of the modified-couple stress theory, is highly significant than that caused by the surface energy effect.

1. Introduction

Every physical theory possesses a certain domain of applicability outside which it fails to predict the physical phenomena with reasonable accuracy, Eringen [1]. For each theory, the domain of application defines the level of the considered constituents and the appropriate processes of interactions between these constituents. The components below this level would not be accounted for and consequently, the interaction process between these components and the other ones would be avoided also. As an obvious example, for a macro-scale body the surface component of the body is very small relative to the volume of the solid. Thus, we can neglect the surface as component of the continuum and focus our attention only on the bulk solid. For a tiny body the surface is very comparable to the bulk volume. Therefore, it should be taken into consideration and deserves to pay a considerable attention to its characteristics and the processes of interactions with the bulk of the continuum.

The same issue can be observed when we study the mechanical deformation of a macro-scale elastic continuum. In this case, it will be sufficient to investigate the behavior on the level of particles as already happened in the classical continuum mechanics theories, Truesdell and Noll [2]. On the contrary, for nano-scale systems we have to deal with the atomic discrete nature of the system. Thus, we have to concern primarily with the level of microstructure elements and investigate different interaction processes between those elements, Chen et al. [3]. Unfortunately, classical continuum mechanics is explicitly designed to be size-independent, which may call the applicability of classical continuum models on nanostructures into question. Several physical reasons may be ascribed to the breakdown of classical continuum mechanics at nano-scale size, Maraganti and Sharma [4].

The surface of a solid is considered as a region with a negligible thickness which has its own atom arrangement and properties differing from the bulk. Atoms at a free surface experience a different local environment than do atoms in the bulk of a solid material. As a result, the energy associated with these atoms will be different from that of the atoms in the bulk. The excess energy associated with surface atoms is surface free energy. For a solid with large size, such surface free energy is typically neglected because it is associated with only a few layers of atoms near the surface and the ratio of the volume occupied by the surface atoms and the total volume of material of interest is extremely small. However, for small solids with a comparable ratio of surface to

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bulk, the surface free energy effect is most likely significant. This is extremely true for nano-scale materials and structures.

Nevertheless, the presence of surface stress gives rise to a non-classical boundary condition which in combination with the constitutive relation of surface and the equations of classical elasticity forms a coupled system of field equations. This makes the solutions of the corresponding boundary value problem relatively difficult.

A generic mathematical model for the analysis of surface elasticity has been developed by Gurtin and Murdoch [5–8], where the surface stresses depend on deformations. The equilibrium and constitutive relations of the bulk solid are the same as those in classical elasticity, but the boundary conditions must ensure the force balance of the surface object. In Gurtin and Murdoch model, the surface is represented as a single layer combining of an infinite number of material particles as in classical elasticity, neglecting the microstructure of the surface. However, Guo and Zhao [9,10] considered the microstructure of the surface of nanofilms, where the surface consists of multi-layers of relaxed crystals. A lattice model is proposed where the possible bond relaxation of the atom is considered which alter the mechanical properties of the nano film.

Miller and Shenoy [11], developed a simple model based on the surface elasticity theory of Gurtin and Murdoch to determine the size effects on the elastic rigidities of nano-sized structural elements such as bars, beams and plates. Thus as the dimensions of the structure become smaller the presence of surfaces have to be accounted for in the modeling strategy. Most of surface effects, such as surface energy, surface tension and surface relaxation are studied by many investigators [9–16]. The effect of the residual stress-due to-surface tension on the bending behavior of nano-plates is studied by Wang and Zhao [12]. Moreover, the effect of surface relaxation in combining with surface tension on the bending behavior of nanobeams and plates is studied by Guo and Zhao [9,10].

The interactions at microscopic scale are the physical origin of many macroscopic phenomena. The fundamental departure of micro-continuum theories from the classical continuum theories is that the former is either a continuum model embedded with microstructures or a nonlocal model to describe the long-range material interaction, Chen et al. [3].

Any attempt to drop the continuity assumption in a modified theory is bounded to make the analysis extremely difficult and computationally intensive. Therefore there is a need for modified continuum theories that include new measures of deformation, which are length related, such as the curvature tensor. As a consequence, such a theory may also require the introduction of couple stresses, Hadjesfandiari and Dargush [17]. Cosserat and Cosserat [18] were the first to develop a mathematical model to analyze materials with couple stresses. In the original Cosserat theory, the kinematical quantities were the displacement and a material microrotation, which assumed to being independent of the continuum macrorotation.

Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume, in addition to the classical direct and shear forces per unit area. This immediately admits the possibility of asymmetric stress tensor, since shear stress no longer have to be conjugate in order to ensure rotational equilibrium. Recently, Yang et al. [19] modified the classical couple stress theory and proposed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect which is caused by micro-structure.

Jomehzadeh et al. [20] developed a variational model for the vibration analysis of ultra-thin plates using the modified couple-stress theory and on the basis of Hamilton's principle. Tsiatas [21] studied the static bending analysis of isotropic micro-Kirchhoff plates using the modified couple-stress theory and on the basis of the principle of minimum potential energy. Ma et al. [22] developed a non-classical Mindlin plate model using the modified couple-stress theory and on the basis of Hamilton's principle.

On the other side, a general classical thin plate theory including surface effects, which can be used for static and dynamic analysis of plate-like thin film structures, was developed by Lu et al. [23]. The modeling of surface effects is based on the surface elasticity theory developed by Gurtin and Murdoch [5,7] and an additional material length scale parameter is determined. Moreover, Shaat et al. [13–15] developed a size-dependent model to study the static bending of Mindlin functionally graded plates incorporating surface energy effects based on Gurtin and Murdoch theory considering effects of surface tension.

The present study is focused on the presentation of a new Kirchhoff nanoplate model, based on the modified couple-stress theory of Yang et al. [19], and taking into account the surface energy and surface tension effects by using the surface elasticity theory of Gurtin and Murdoch. Classical Newtonian approach is used to derive the differential form of the equilibrium equations of the generalized Kirchhoff nanoplate.

The rest of the paper is organized as follows. In Section 2, the formulation of the equilibrium equations for the non-classical Kirchhoff plate model is developed using the modified couple stress theory (Yang et al. [19], Park and Gao [24]) and the surface elasticity theory of Gurtin and Murdoch [5,7]. Constitutive equations of the bulk and surface layer materials in addition to the kinematic equations of the Kirchhoff plate are presented in Section 3. Moreover, a length scale parameter, in the context of the modified couple stress theory, is presented to capture the size effect in nano-plates. Based on the equilibrium equations, constitutive relations and the kinematic equations; the equilibrium equations in terms of deflection are derived in the end of Section 3. To demonstrate the new proposed model, a simply supported plate problem is solved in Section 4, by applying the equilibrium equations derived in Section 3. Some numerical results are presented in Section 5 to show both the microstructure and surface energy effects on the bending rigidity of the plate. In addition, a parametric study is given to present the effect of surface parameters and the effect of the length scale parameter, mentioned in Section 3, on the bending behavior of simply supported Kirchhoff plates.

2. Equilibrium equations of the modified couple-stress plate theory including surface effects

The formulation of the equilibrium model for Kirchhoff plate including surface effects, in the framework of the modified couple stress theory, will be presented throughout this section. Surface energy and surface tension effects are handled through Gurtin and Murdoch model neglecting the microstructure of the surface. This formulation is developed on the basis of the classical Newtonian continuum mechanics, Reddy [25].

Consider an ultra-thin rectangle plate with uniform thickness h. A Cartesian coordinate system $x_i (i=1,\ 2,\ 3)$ is introduced so that the axes x_1 and x_2 are lying in the mid-plane of the plate, and the upper and lower surfaces S^+ and S^- of the plate are defined by $x_3 = \pm h/2$, respectively (see Fig. 1).

The differential form of the equilibrium equations for a size-dependent continuum, based on the modified couple-stress theory, is given by

$$\sigma_{ji,j} + f_i = 0 \tag{1}$$

$$\mu_{ji,j} + e_{ijk}\sigma_{jk} + C_i = 0 \tag{2}$$